Effect of torque-velocity relationship on manipulability for robot manipulators

Tetsuyou Watanabe
Assistant Professor
School of Mechanical Engineering
College of Science and Engineering
Kanazawa University
Kakuma-machi, Kanazawa, 920-1192
Email: te-watanabe@ieee.org

ABSTRACT

This paper presents novel manipulability analysis for robotic manipulators, taking the effect of generating joint torques on generable joint velocities and vice versa into consideration. The conventional manipulability is analysis in velocity domain and cannot concern force effect such as gravity of payload and external forces exerted on the endeffector. Gravitational force has been regarded that it just changes the origin of the manipulability ellipsoid expressing the set of generable tip velocities, and its evaluation (its volume) does not change. However, if robot grasps a heavy object, the robot cannot move with the same speed as the case of grasping a light object, because the power of the robot is limited. It indicates that the robot performance evaluation by conventional manipulability has serious problem that the force effect cannot be included. The power of the robot is determined by the operation range of every actuator, which tells us the relationship between generating torque/velocity and addable velocity/torque. Then, this paper presents novel manipulability analysis which can take the force effect into consideration, based on the torque-velocity relationship. This analysis shows that manipulability is influenced by payload, gravitational force and external forces.

1 INTRODUCTION

Manipulability is a well known concept to evaluate the performance of robotic manipulator [1]. It is defined as the set of generable velocities of endeffector in the task space when the set of generable joint velocities is given. When the given set of joint velocities is a unit ball, the set of the endeffector velocities becomes an ellipsoid. The ellipsoid is called manipulability ellipsoid. Its volume is a quality measure to evaluate the performance of manipulators in velocity domain, and called manipulability measure. Based on the manipulability, many quality measures such as condition number are proposed [1].

However, manipulability can not include the effects of gravity of link, payload and external forces. In actual systems, manipulators are sometimes used under influence of gravity. If we do not concern the effect of gravity in such a case, we will get the result that the performance when moving a heavy object is the same as the performance when moving a light object. Then the performance evaluation becomes invalid. On the other hand, in some tasks such as tracing, manipulator is required to keep contact with environment. In such a case, we should evaluate the performance, concerning the effect of the contact forces. However, the conventional manipulability analysis can not include such a force effect. For example, gravitational force has been regarded that it just changes the origin of the manipulability ellipsoid expressing the set of generable tip velocities, and its evaluation (its volume) does not change.

Concerning described the above, this paper presents a new manipulability based approach to evaluate performance of manipulator in velocity domain, which can include force effect. For the purpose, we use an operation range (see Fig.1) of actuator attached with every joint of robot (operation range is originally used for motor selection). The operation range provides not only the information about how much of magnitudes of torque and velocity the actuator can stably generate, but also the relationship between generating velocity/torque and addable torque/velocity. If using this relationship, we can take the magnitude of required joint torque into consideration when evaluating the generable tip/endeffector velocities.

The contributions of this paper are as follows.

1) We derive a manipulability convex polyhedron corresponding to the conventional manipulability ellipsoid, taking force effects into consideration.
2) We derive 3 kinds of corresponding manipulability measures: the volume of the manipulability convex polyhedron, the magnitude of generable tip velocity which is generable in any direction, and the maximum magnitude of generable tip velocity.
3) We also evaluate how much magnitude of tip forces the robot can generate, considering the effect of generating/required tip velocities.

First, we derive required joint torques to compensate gravity effect and external forces, and keep configuration/pose of manipulator. Using the required joint torques and the operation range, we derive usable (actually generable) joint velocities. Using the usable joint velocities, we derive a set of generable tip/endeffector velocities. The set is described by a convex polyhedron. Based on this set, we derive the new manipulability measures described the above.

In some tasks such as assembling, drilling and deburring, the tip of the manipulator is required to contact with environment such as a table, and to generate tip force. To include the effect of the tip force, we introduce required external force set (REFS) [2,3] which is defined as a set of external forces required to be compensated. Based on the REFS, we derive required joint torque to compensate any force contained in REFS, and derive a set of generable endeffector/tip velocities and the corresponding manipulability measures. In this paper, we present an approach when REFS is given as a convex polyhedron or ellipsoid.

We also present a way to evaluate the generable tip forces. For example, if we want to grind a cup with a certain constant speed, large contact force for keeping contact would be preferable. Then, to include the effect of endeffector/tip velocities, we introduce a required velocity set (RVS) defined as a set of required endeffector/tip velocities to be generated in a given task. Based on the RVS, we derive required joint velocity to generate any endeffector/tip velocity contained in RVS, and derive a set of generable tip forces and the corresponding manipulability measures.

Manipulability analysis was extended to various fields such as robotic hand [4–6] and musculo-skeletal analysis [7]. However, there is no manipulability research for manipulator from the viewpoint of power to include both force and velocity effects.

2 Manipulability taking force effects into consideration

In this paper, we consider a serial n link manipulator with n joints. A payload is attached to the tip of the manipulator. In order to evaluate the effect of the payload, external forces and gravity of manipulator, we introduce the following set:

Joint Torque-velocity Pair Set (TVS): The set of generable joint torques and velocities at each joint, given by the corresponding actuator and gear specifications, is named joint torque-velocity pair set (TVS).

The specification for actuators (operation range) is usually given with respect to the absolute values of torque and velocity. We express generable maximum absolute values of joint torque and velocity with $|\tau_{i_{max}}|$ and $|\dot{q}_{i_{max}}|$. Let $|\dot{q}_{i_{Umax}}|(\geq 0)$ be the usable maximum absolute value of joint velocity, determined by currently generating joint torque. Similarly, let $|\tau_{i_{Umax}}|$ (≥ 0) be the usable maximum absolute value of joint torque, determined by currently generating joint velocity. We describe this relationship with the following functions.

$$|\dot{q}_{i_{Umax}}| = \xi_i^{\tau\to\dot{q}}(|\tau_i|), \tag{1}$$

$$|\tau_{i_{Umax}}| = \xi_i^{\dot{q}\to\tau}(|\dot{q}_i|) = (\xi_i^{\tau\to\dot{q}})^{-1}(|\dot{q}_i|). \tag{2}$$

This function $\xi_i^{\tau\to\dot{q}}$ can be derived from the actuator and gear specifications. For example, if using DC motor under the constant nominal voltage, $\xi_i^{\tau\to\dot{q}}$ can be derived by utilizing maximum speed under the voltage, maximum torque, and torque-speed constant. An example of TVS is the area surrounded by the rigid lines in Fig.1, and $\xi_i^{\tau\to\dot{q}}$ gives the rigid lines. In this case, the TVS is convex. If we can not get the information about torque-speed constant, we use power. Let ψ_i be the constant power for the evaluation. Then, $\xi_i^{\tau\to\dot{q}}$ can be expressed by

$$\xi_i^{\tau\to\dot{q}} = \begin{cases} \psi_i/|\tau_i| & \psi_i \leq |\tau_i||\dot{q}_{i_{max}}| \\ |\dot{q}_{i_{max}}| & otherwise \end{cases}. \tag{3}$$

Then, we consider the following problem:

Problem 1: Suppose that TVS is given. In this case, derive the set of generable endeffector/tip velocities, $\mathcal{S}_{ev}$, and corresponding (manipulability) measures.

First, we consider the relationship between joint velocity and endeffector/tip velocity. Let joint angle vector be $q = [q_1\ q_2\ \cdots\ q_n]^T$, and the tip position of the manipulator $r \in \mathcal{R}^D$ where $D = 3$ in two dimensional space and $D = 6$ in three dimensional space. Then, the relationship is expressed by:

$$\dot{r} = J\dot{q}, \tag{4}$$

where $J \in \mathcal{R}^{D\times n}$ denotes Jacobian matrix. From (4), we get

$$\dot{q} = J^{+}\dot{r} + Ek_q, \tag{5}$$

where J^{+} denotes the psedo-inverse matrix of J, E denotes an orthogonal matrix whose columns form bases of the null space of J, and k_q denotes an arbitrary vector.

From (4) and the principle of virtual work, the following relation is obtained:

$$\tau_q = J^T f, \tag{6}$$

where $\tau_q \in \mathcal{R}^n$ denotes joint torque vector corresponding to $\dot{q}$, and $f \in \mathcal{R}^D$ denotes tip force corresponding to $\dot{r}$.

Let the gravity term of the manipulator be $g_m = \partial \sum_i {U_i}^T / \partial q$ where U_i is the potential energy of the ith link of the manipulator due to the gravitational force. Let also the mass of the payload be m_p, gravitational acceleration vector be g, and the other external force exerted on the tip be f_{ex}. Then, overall joint torque, $\tau = [\tau_1\ \tau_2\ \cdots\ \tau_n]^T \in \mathcal{R}^n$, is expressed by

$$\tau = g_m - J^T m_p g - J^T f_{ex}. \tag{7}$$

where $-J^T f_{ex}$ is the term for compensating f_{ex}.

If $f_{ex} = o$, from (1), the usable maximum joint velocity vector, $\dot{q}_{Um} = [\dot{q}_{1_{Umax}}\ \dot{q}_{2_{Umax}}\ \cdots\ \dot{q}_{n_{Umax}}]^T \in \mathcal{R}^n$ is expressed by

$$\dot{q}_{Um} = \{ \begin{bmatrix} \dot{q}_{1_{Umax}} \\ \dot{q}_{2_{Umax}} \\ \vdots \\ \dot{q}_{n_{Umax}} \end{bmatrix}, \ |\dot{q}_{i_{Umax}}| = \xi_i^{\tau\to\dot{q}}(|\tau_i|),\ \tau = g_m - J^T m_p g\}. \tag{8}$$

Then, from (4), and (8), $\mathcal{S}_{ev}$ is expressed by

$$\begin{aligned} \mathcal{S}_{ev} = \{\dot{r}_w | \dot{r}_w = MJ\dot{q},\ -|\dot{q}_{i_{Umax}}| \le \dot{q}_i \le |\dot{q}_{i_{Umax}}|, \\ |\dot{q}_{i_{Umax}}| = \xi_i^{\tau\to\dot{q}}(|\tau_i|),\ \tau = g_m - J^T m_p g\}. \end{aligned} \tag{9}$$

Since $\dot{q}_{Um}$ is uniquely determined and the other equations and inequalities constituting $\mathcal{S}_{ev}$ are linear, $\mathcal{S}_{ev}$ is a convex polyhedron. Therefore, we call $\mathcal{S}_{ev}$ joint torque-velocity pair based manipulability polyhedron (TVMP). Here we will consider the following 3 manipulability measures: 1) α_v: the volume of $\mathcal{S}_{ev}$, 2) α_{max}^{all}: the maximum tip velocity magnitude which is generable in any arbitrary direction, and 3) α_{max}: the maximum magnitude of generable tip velocity. The first one is typical measure. The second one corresponds to the radius of the hypersphere inscribed in $\mathcal{S}_{ev}$. The third one corresponds to the radius of the hypersphere circumscribed in $\mathcal{S}_{ev}$. Note that in order to treat translational and rotational velocities at the same time, we use the weight matrix, M, such that $\dot{r}_w = M\dot{r}$ can appropriately evaluate the (weighted) endeffector/tip velocity in every direction: for example, every direction can have the same unit. Note also that the choice of M affects the evaluation, and then we should carefully choose M such that the evaluation can be valid and reasonable.

$\mathcal{S}_{ev}$ can be expressed using its extremes:

$$\mathcal{S}_{ev} = \{\dot{r}_w | \dot{r}_w = \Sigma_{i=1}^{n_{s_{ev}}} \lambda_i \dot{r}_{vi},\ \Sigma_{i=1}^{n_{s_{ev}}} \lambda_i = 1,\ \lambda_i \ge 0\}, \tag{10}$$

where $\dot{r}_{vi}$ denotes the vertex of $\mathcal{S}_{ev}$ and $n_{s_{ev}}$ denotes the number of the vertices. This expression is called V-representation [8]. Generally, this transformation can be done by a programming method such as double description method [9]. However, since every $|\dot{q}_{iU_{max}}|$ is unique, every V-representation for $\dot{q}_i$ can be easily derived as follows. The generable joint velocity set for every joint is expressed by

$$\{\dot{q}_i = -\eta_1|\dot{q}_{iU_{max}}| + \eta_2|\dot{q}_{iU_{max}}|,\ \Sigma_{j=1}^{2}\eta_j = 1,\ \eta_j \geq 0\} \tag{11}$$

Since the generable joint velocity vector set is given by the direct sum of the generable joint velocity sets for all joints, it can be expressed by

$$\{\dot{q} = \Sigma_{i=1}^{2^n}\lambda_i \dot{q}_{vi},\ \Sigma_{i=1}^{2^n}\lambda_i = 1,\ \lambda_i \geq 0\}$$

where $\dot{q}_{vi}$ denotes the vertices of the set. Then, the V-representation for $\mathcal{S}_{ev}$ is expressed by

$$\mathcal{S}_{ev} = \{\dot{r}_w | \dot{r}_w = \Sigma_{i=1}^{n_{sev}}\lambda_i \dot{r}_{vi},\ \Sigma_{i=1}^{n_{sev}}\lambda_i = 1,\ \lambda_i \geq 0\}, \tag{12}$$

where

$$\dot{r}_{vi} = MJ\dot{q}_{vi}, \quad n_{s_{ev}} = 2^n.$$

Let $\int$ be a simplex in d dimensional space, and v_{si} $(i = 0, \cdots, d)$ be its vertex. In this case, the volume of the simplex is given by

$$V(\int) = \frac{|\,[v_{s1} - v_{s0}\ v_{s2} - v_{s0}\ \cdots\ v_{sd} - v_{s0}]\,|}{d!}. \tag{13}$$

Therefore, if polyhedron $\mathcal{S}_{ev}$ can be decomposed into simplices, we can calculate the volume of $\mathcal{S}_{ev}$. One of the famous methods for the decomposition is triangulation method using delaunay triangulation [8, 10]. Then, letting $\int_i$ $(i = 1, \cdots, n_{s_{sev}})$ be the decomposed simplices of $\mathcal{S}_{ev}$, the volume of $\mathcal{S}_{ev}$ (TVMM), α_v, is calculated by

$$\alpha_v = \Sigma_{i=1}^{n_{s_{sev}}} V(\int_i). \tag{14}$$

Note that qhull [11] can also compute the volume of the convex polyhedron (α_v).

Next, we consider the maximum tip velocity magnitude which is generable in any arbitrary direction, α_{max}^{all}. From (12), we will derive H-representation (expression by half planes) of $\mathcal{S}_{ev}$.

$$\mathcal{S}_{ev} = \{\dot{r}_w | A_w \dot{r}_w \leq b_w\}. \tag{15}$$

where A_w and b_w are constant matrix and vector resulted from the transformation. Cdd librarly [8] or qhull [11] can be used to derive this H-representation. If letting $A_w = \mathrm{col}[a_{wi}^T]$ and $b_w = \mathrm{col}[b_{wi}]$ where col denotes a column vector or matrix formed by the following elements, every $a_{wi}^T \dot{r}_w \leq b_{wi}$ expresses every face of $\mathcal{S}_{ev}$. The distance between the face $a_{wi}^T \dot{r}_w \leq b_{wi}$ and the origin can be computed as follows.

$$d_{wi} = b_{wi}/|a_{wi}|.$$

Therefore, α_{max}^{all} can be obtained by

$$\alpha_{max}^{all} = \min_i d_{wi} \tag{16}$$

Lastly, we consider the maximum magnitude of generable tip velocity α_{max}. From (12), it can be obtained by

$$\alpha_{max} = \max_i |\dot{r}_{vi}|. \tag{17}$$

From (5), (9) and (15), its alternative way is to solve the following convex quadratic programming problem.

$$\begin{aligned} \alpha_{max} = & \max_{\dot{r},k_q} \quad \dot{r}^T M^T M \dot{r} = \dot{r}_w^T \dot{r}_w \\ \text{subject to} & \quad -\dot{q}_{Um} \leq J^+ \dot{r} + E k_q \leq \dot{q}_{Um} \end{aligned}$$

or

$$\begin{aligned} \alpha_{max} = & \max_{\dot{r},k_q} \quad \dot{r}^T M^T M \dot{r} = \dot{r}_w^T \dot{r}_w \\ \text{subject to} & \quad A_w \dot{r}_w \leq b_w \end{aligned} \tag{18}$$

Remark that the required joint torques to compensate payload, link gravity, and external forces determine the range of feasible joint velocities. If the required joint torques are large (close to their maximum), the range of feasible joint velocities and the generable tip/endeffector velocities would be small. If the required joint torques exceed their maximum, the robot cannot keep its configuration and then the robot cannot generate tip/endeffector velocities.

2.1 When tip is constrained

Consider the case when a task for manipulator is given, and its tip is constrained. In such a case, We need to generate tip forces, for example, to keep the contact. In order to consider the effect of the tip forces, we introduce the following Required External Force Set (REFS):

Required External Force Set (REFS): The set of the endeffector's external force required to be compensated is named required external force set (REFS). We suppose that the we can cope with any external force if any arbitrary external force contained in REFS can be compensated.

REFS is assumed to be given by a convex polyhedron or an ellipsoid:

$$S_{ref}^{pol} = \{f_{ex} | f_{ex} = \Sigma_{i=1}^{n_{ref}} \kappa_i f_{v_i},\ \Sigma_{i=1}^{n_{ref}} \kappa_i = 1,\ \kappa_i \geq 0\}, \tag{19}$$

$$S_{ref}^{elip} = \{f_{ex} | f_{ex}^T M_f^T M_f f_{ex} \leq 1\}. \tag{20}$$

Here, f_{v_i} denotes the vertex of the convex polyhedron, n_{ref} denotes the number of the vertices, M_f denotes a weight matrix. Note that f_{ex} can have moment component. M_f is added similarly to M in (9) such that every component of f_{ex} can be equally treated: for example, every direction can have the same unit, or the magnitude of external forces can be normalized. Note also that M_f has to be selected such that the choice of M_f can be consistent with the choice of M. One of the ways might be $M_f = M^T$. Another way will be described in the section 3 for numerical examples.

Using this REFS, we consider the following modified problem:

Problem 2: Suppose that TVS and REFS are given. In this case, find the set of generable endeffector velocities, S_{ev}, such that the velocity contained in S_{ev} can be generated even if any f_{ex} contained in REFS is exerted on the manipulator. In addition, find the corresponding (manipulability) measures: α_v, α_{max}^{all} and α_{max}.

First, we consider the case when REFS is given as a convex polyhedron S_{ref}^{pol} (19). We derive minimum required joint torque to compensate any f_{ex} contained in REFS. If we can compensate every f_{v_i}, we can compensate any f_{ex} contained in REFS. Therefore, from (7), the minimum required joint torque $\tau_r = [|\tau_{r1}| \ \cdots \ |\tau_{rn}|]^T$ can be obtained by the following problem.

$$\begin{aligned} & \min \quad \tau_r \\ & |\tau_{rc_\iota}| \leq \tau_r \\ & \tau_{rc_\iota} = g_m - J^T m_p g - J^T f_{v_\iota} \\ & \quad (\iota = 1, \cdots, n_{ref}). \end{aligned} \tag{21}$$

Since τ_{rc_ι} is constant for ι, we calculate τ_{rc_ι} for every ι and just compare its magnitude at every joint. Then, we can get the the minimum required joint torque τ_r.

Next, we consider the case when REFS is given as an ellipsoid $\mathcal{S}_{ref}^{elip}$ (20). Let $\hat{f}_{ex} = M_f f_{ex}$. Then, from (7), the minimum required joint torque τ_r has to satisfy the following condition to compensate the external force $\hat{f}_{ex}$

$$A_{f1}\hat{f}_{ex} + A_{f2}\tau_r \le b_f \tag{22}$$

where

$$A_{f1} = \begin{bmatrix} -J^T M_f^{-1} \\ J^T M_f^{-1} \end{bmatrix}, A_{f2} = \begin{bmatrix} -I \\ -I \end{bmatrix}, \; b_f = \begin{bmatrix} -g_m + J^T m_p g \\ g_m - J^T m_p g \end{bmatrix}.$$

Let $A_{f1} = \mathrm{col}[a_{f1_i}^T]$. If we had already obtained τ_r, (22) could express a convex polyhedron for $\hat{f}_{ex}$, and a_{f1_i} could express the normal of the face of the convex polyhedron. Generally, the distance between the origin and a hyperplane $a^T x = b$ is given by $b/|a|$. Therefore, if holding the following conditions, we can compensate any arbitrary $f_{ex} \in \mathcal{S}_{ref}^{elip}$.

$$\mathrm{col}[|a_{f1_i}|] + A_{f2}\tau_r \le b_f. \tag{23}$$

Here, taking into consideration that A_{f2} is constructed by identity matrices, from (23), we can get

$$\tau_r \ge \hat{b}_f \tag{24}$$

where $\hat{b}_f$ is the vector resulted from the transformation of the inequality, and indicates the minimum required joint torque.

Now, we have minimum required joint torque. From (1), we can get the usable maximum joint velocity vector, $\dot{q}_{Um}$. Then, we can use the same way as the discussion from (10) to (18), in order to get $\mathcal{S}_{ev}$, and its corresponding manipulability measures: α_v, α_{max}^{all} and α_{max}.

2.2 Force evaluation

In tasks such as deburring and drilling, manipulator moves with its endeffector contacting with environment. In this case, it is also important to evaluate how magnitude of tip/endeffector force can be generated and in which directions the force can be generated. It is also important to consider manipulator motion, namely, manipulator speed. To take the effect of manipulator motion into consideration, we define the following set:

Required Velocity Set (RVS): The set of endeffector's velocities required to be generated is named required velocity set (RVS). We suppose that any endeffector velocity required in a given task can be generated if any endeffector velocity contained in RVS can be generated.

Using RVS, we will evaluate tip/endeffector forces in the case when manipulator moves with a velocity contained in RVS. RVS is assumed to be given by a convex polyhedron or an ellipsoid:

$$\mathcal{S}_{rvs}^{pol} = \{\dot{r} | \dot{r} = \Sigma_{i=1}^{n_{rv}} \lambda_i \dot{r}_{v_i},\, \Sigma_{i=1}^{n_{rv}} \lambda_i = 1,\, \lambda_i \ge 0\}, \tag{25}$$

$$\mathcal{S}_{rvs}^{elip} = \{\dot{r} | \dot{r}^T M^T M \dot{r} \le 1\}. \tag{26}$$

where $\dot{r}_{v_i}$ denotes the vertex of RVS and n_{rv} denotes the number of vertices of RVS. M denotes a weight matrix, for example, to consider the difference between the units of translational and rotational velocities, or to normalize the magnitude of the velocity.

Using this RVS, we consider the following problem:

Problem 3: Suppose that TVS and RVS are given. In this case, derive the set of generable tip/endeffector forces $\mathcal{S}_{ef}$ such that the force contained in $\mathcal{S}_{ef}$ can be generated even if any tip/endeffector velocity $\dot{r}$ contained in RVS is generating. In addition, find the corresponding (force manipulability) measures: β_v (the volume of $\mathcal{S}_{ef}$), β_{max}^{all} (the maximum magnitude of tip/endeffector force which is generable in any arbitrary direction) and β_{max} (the maximum magnitude of generable tip/endeffector force).

First, we consider the case when RVS is given as a convex polyhedron $\mathcal{S}_{rvs}^{pol}$ (25). We derive required joint velocity to generate any tip velocity $\dot{r}$ contained in RVS. If we can generate every $\dot{r}_{v_i}$, we can generate any $\dot{r}$ contained in RVS. Therefore, from (5), the required joint velocity $\dot{q}_r = [|\dot{q}_{r1}| \cdots |\dot{q}_{rn}|]^T$ holds the following constraints.

$$-\dot{q}_r \le J^+ \dot{r}_{v_\iota} + E k_q \le \dot{q}_r \quad (\iota = 1, \cdots, n_{rv}). \tag{27}$$

Next, we consider the case when RVS is given as an ellipsoid $\mathcal{S}_{rvs}^{elip}$ (26). From (5), the required joint velocity $\dot{q}_r$ has to satisfy the following condition to generate $\dot{r}$ (or $\dot{r}_w = M\dot{r}$):

$$A_{r1}\dot{r}_w + A_{r2}k_q + A_{r3}\dot{q}_r \leq o \tag{28}$$

where

$$A_{r1} = \begin{bmatrix} J^+M^{-1} \\ -J^+M^{-1} \end{bmatrix}, \quad A_{r2} = \begin{bmatrix} E \\ -E \end{bmatrix}, \quad A_{r3} = \begin{bmatrix} -I \\ -I \end{bmatrix}.$$

Now, we remove k_q from (28). First, we transform (28) to its V-representation.

$$x_r = \Sigma_{i=1}^{n_{Ar}} \lambda_i x_{rvi},\ \Sigma_{i=1}^{n_{Ar}} \lambda_i = 1,\ \lambda_i \geq 0$$

where

$$x_r = \begin{bmatrix} \dot{r}_w \\ k_q \\ \dot{q}_r \end{bmatrix},$$

and x_{rvi} denotes the vertex and n_{Ar} denotes the number of the vertices. We eliminate the terms of k_q by multiplying the following matrix from the left side

$$\begin{bmatrix} I & O & O \\ O & O & I \end{bmatrix}.$$

After that, we transform it to its H-representation.

$$\hat{A}_{r1}\dot{r}_w + \hat{A}_{r3}\dot{q}_r \leq \hat{b}_r \tag{29}$$

This two transformations can be done by cdd library [8].

Let $\hat{A}_{r1} = \mathrm{col}[\hat{a}_{r1_i}^T]$. Now, if $\dot{q}_r$ had been given, (29) could express a convex polyhedron for $\dot{r}_w$, and $\hat{a}_{r1_i}$ could express the normal of the face of the convex polyhedron. Therefore, similarly to the previous subsection, if holding the following conditions, we can generate any arbitrary $\dot{r} \in \mathcal{S}_{rv}^{elip}$.

$$\mathrm{col}[|\hat{a}_{r1_i}|] + \hat{A}_{r3}\dot{q}_r \leq \hat{b}_r. \tag{30}$$

Now, we have H-representation for $\dot{q}_r$, similarly to (27).

Here, we will assume that the relationship between joint torque and velocity (2) can be represented by

$$A_{tq_{1i}}|\tau_{i_{Umax}}| + A_{tq_{2i}}|\dot{q}_{ri}| \leq b_{tq_i} \tag{31}$$

where $A_{tq_{1i}}$ $A_{tq_{2i}}$ and b_{tq_i} are the matrices and vector expressing the boundary of the region. If aggregating this relationship for all joints, we get

$$A_{tq}\begin{bmatrix} \tau_{Um} \\ \dot{q}_r \end{bmatrix} \leq b_{tq} \tag{32}$$
$$A_{tq} = \left[\,\mathrm{diag}[A_{tq_{1i}}]\ \mathrm{diag}[A_{tq_{2i}}]\,\right],\ b_{tq} = \mathrm{col}[b_{tq_i}].$$

Then, with the same way as the way of form (28) to (29), from (32) and (27) if using $\mathcal{S}_{rv}^{pol}$ or (30) if using $\mathcal{S}_{rv}^{elip}$, we get the usable range of joint torques

$$A_{Utau}\tau_{Um} \leq b_{Utau} \tag{33}$$

Then, from (7) and (33), $\mathcal{S}_{ef}$ is expressed by

$$\begin{aligned}\mathcal{S}_{ef} = \{\hat{f}_{ex}| -\tau_{Um} \leq \tau_c \leq \tau_{Um},\ A_{Utau}\tau_{Um} \leq b_{Utau}\\ \tau_c = g_m - J^T m_p g - J^T M_f^{-1}\hat{f}_{ex}\}.\end{aligned} \tag{34}$$

Here, $\mathcal{S}_{ev}$ is expressed by H-representation. It can be transformed to V-representation with cdd library [8].

$$\mathcal{S}_{ef} = \{\hat{f}_{ex}|\hat{f}_{ex} = \Sigma_{i=1}^{n_{s_{ef}}}\lambda_i\hat{f}_{ex_{vi}},\ \Sigma_{i=1}^{n_{s_{ef}}}\lambda_i = 1,\ \lambda_i \geq 0\}. \tag{35}$$

where $\hat{f}_{ex_{vi}}$ denotes the vertex and $n_{s_{ef}}$ denotes the number of the vertices.

If J has row full rank, we get

$$\hat{f}_{ex} = (J^T M_f^{-1})^+(-\tau_c + g_m - J^T m_p g). \tag{36}$$

In this case, we have alternative way for the transformation for the V-representation of $\mathcal{S}_{ef}$. First, we will get the V-representation of the range for τ_c using (33) and $-\tau_{Um} \leq \tau_c \leq \tau_{Um}$. Then, from (36), we can get V-representation for $\mathcal{S}_{ef}$.

The manipulability measures, β_v, β_{max} and β_{max}^{all} can be obtained by the same way as the deviation of α_v, α_{max} and α_{max}^{all} (see from (12) to (18)).

3 Numerical examples

In order to verify our approach, we show some numerical examples. For the convenient, we only considered translational directions in task space. First, we considered Problem 1 for 2-link planar manipulator shown in Fig.2. Base position was located at origin. The length of every link was set to 0.1[m] and the gravity center of every link was set to the geometric center of the link. Mass of every link was set to 0.051[kg]. Gravitational acceleration vector was set to $[0\ -9.8]^T$[m/s^2]. We used the operation range shown in Fig.1. The actuators are all the same.

Here, we considered the case when the tip position changes from origin to $[0.2\ 0]^T$ as shown in Fig.2. We calculated the proposed new manipulability polyhedron (TVMP) and measures, α_v, α_{max}^{all}, and α_{max} when mass of the payload attached to the tip of the manipulator, m_p, is 0.0[kg], 0.1[kg], 0.2[kg], 0.3[kg] and 0.4[kg]. For the comparison, we also calculated original manipulability ellipsoid and measure.

Fig.2 (a)∼(f) and Fig.3 (a)∼(c) show the results. Note that as it can be seen from Fig.3, when $m_p = 0.4$ and x coordinate of the tip position is around 0.2, α_v and α_{max}^{all} are 0 (and α_{max} becomes around 0.2). It means that manipulator can not generate endeffector velocity in arbitrary directions due to the large mass of the payload. In other words, endeffector velocity can be generated in only the specified direction as shown in Fig. 2 (f). Then, α_v, α_{max}^{all} and α_{max} decreases largely. Comparing Fig.2 (a) with Fig.2 (b)∼(f), it can be seen that the direction of maximum (and minimum) generable velocity is different. Also, it can be seen that with the increase of mass of payload, maximum generable velocity becomes smaller. Especially, the velocity in y direction in which gravitational force is applied decreases. From Fig.3, it can be seen that the tip position whose α_v is maximum becomes closer to that for original manipulability measure with decrease of the payload mass. Note that we will not compare the magnitudes of new and original manipulability measures since the given set of generable joint velocities is different although maximum generable joint velocity is the same. The tip positions for maximum α_v, α_{max}^{all}, and α_{max} are all different. With the increase of mass of the payload, all manipulability measures, α_v, α_{max}^{all}, and α_{max} become smaller. α_{max} increase monotonously with the incase of x coordinate of the tip position while the other measures increases firstly but decrease after reaching their maximums. It is considered to be related with the shape of the TVMP. These results indicate that we can not evaluate the effect of the payload mass by the original manipulability while the proposed manipulability and its measures can evaluate the effect.

Next, we considered Problem 2 for the manipulator shown in Fig.4. Its tip moved on the spherical environment whose center is $[0\ 0\ -0.1/\sqrt{2}]$ and whose radius is 0.1. Note that in Fig.4, the part ($z \geq 0$) of the environment is only shown. We call the joints and the links, joint 1, 2, 3, 4, and link 1, 2, 3, 4, respectively, in the order of closeness to the base side. The lengths of link 1 and 2 were set to be 0. The lengths of link 3 and 4 were set to be 0.1[m] and the gravity center of each link was set to the geometric center of the link. Masses of link 3 and 4 were set to 0.051[kg]. Mass of the payload was set to 0.1[kg]. Base position was located at $[-0.1\ 0\ 0]^T$. We used the operation range shown in Fig.1. The actuators are all the same.

Here, we considered to move the tip in parallel with x axis, contacting with the environment. Let θ be the angle between z axis and outward normal vector of the environment at the contact point between the environment and the tip. The manipulator moved the tip such that θ changes from 0 to $\pi/4$.

The maximum contact force was supposed to be 1[N], and the maximum frictional coefficient was supposed to be 0.3. Under these suppositions, REFS was set as follows:

$$\begin{aligned} \mathcal{S}_{ref}^{pol} =& \{f_{ex} | f_{ex} = RE_n f_n,\ f_n = \kappa_1 + \kappa_2 \cdot 0, \\ & \qquad \Sigma_{i=1}^{2} \kappa_i = 1,\ \kappa_i \geq 0\}, \\ \mathcal{S}_{ref}^{elip} =& \{f_{ex} | f_{ex} = RE_t f_t,\ f_t^T f_t \leq 0.3^2 f_n^2\}, \end{aligned}$$

where

$$R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\theta) & \sin(\theta) \\ 0 & -\sin(\theta) & \cos(\theta) \end{bmatrix},\ E_t = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix},\ E_n = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.$$

We used $\mathcal{S}_{ref}^{pol}$ and $\mathcal{S}_{ref}^{elip}$ together. We derived the constraints (24) for every vertex of $\mathcal{S}_{ref}^{pol}$. Then, we can get the combined problem for $\mathcal{S}_{ref}^{pol}$ and $\mathcal{S}_{ref}^{elip}$. By solving the combined problem, we derived minimum required joint torques. Since the tip moves contacting with the environment, we considered the velocities in the tangential directions at the contact point. We set M as follows.

$$M = RE_t.$$

The target manipulator has redundancy. Therefore, the configuration of manipulator was calculated, letting q_2 be constant. Here, we considered the case when $q_2 = 0, -\pi/16, -\pi/8, -3\pi/16, -\pi/4$.

Fig.5 and Fig.6 show the results. It can be seen that when θ is small (around 0), the smaller q_2 is, the larger the volume of TVMP (α_v) is. On the other hand, if θ is large, then, the larger q_2 is, the larger the volume of TVMP (α_v) is. α_{max}^{all} also shows similar tendency. If focusing on the shape of the TVMP when θ is small (around 0), it shows larger anisotropy with the decrease of q_2. On the other hand, if θ is large, then, the larger q_2 is, the larger the anisotropy is. This tendency corresponds to α_{max}. Summarizing, in order to conduct the given task efficiently, the larger θ is, the larger q_2 we should take.

Lastly, we considered Problem 3 for the same manipulator shown in Fig.4. Here, we considered the same task. But, in this case, we suppose that we need to generate a velocity in any arbitrary tangential direction with the maximum magnitude of 0.15[m/s]. Under this condition, we derived generable tip forces. We set RVS as follows

$$\mathcal{S}_{rvs}^{elip} = \{\dot{r} | \dot{r} = 0.15 RE_t \dot{r}_t,\ \dot{r}_t^T \dot{r}_t \leq 1\}.$$

We set M_f as follows.

$$M_f = R \left[0.3E_t\ E_n \right].$$

Here we took the friction effect into consideration.

Fig.7 and Fig.8 show the results. It can be seen that the volume of force manipulability polyhedron (β_v) becomes larger with the decrease of q_2 while becomes smaller with the increase of θ. If focusing on the shape of the force manipulability polyhedron, the smaller q_2 is, the larger anisotropy we get. This tendency is also true of β_{max}. Around $\theta = 0.2\pi \sim 0.25\pi$, β_{max} for small q_2 becomes smaller with the increase of θ while that for large q_2 becomes larger with the increase of θ. One of the reasons would be that with the decrease of q_2, the contact tangential surface becomes closer to the plane constructed by links 3 and 4 (the plane where links 3 and 4 move). β_{max}^{all} becomes smaller with the increase of θ. The ratio of the decrease becomes smaller with the increase of q_2. Therefore, when θ is small, the smaller q_2 is, the larger β_{max}^{all} is, while around $\theta = 0.2\pi \sim 0.25\pi$, the larger q_2 is, the larger β_{max}^{all} is. Summarizing, if we want to generate large contact forces in this task, we should keep q_2 small. If we want to uniformly generate large contact forces in this task, we should keep q_2 small if θ is small and keep q_2 large if θ is large, especially θ is around $0.2\pi \sim 0.25\pi$.

Remark that here we showed that our proposed approach can evaluate generable tip/endeffector velocities and forces even when the tip is constrained by an environment and applying tip forces are required, and the directions of generable tip velocities are limited. The performance evaluation in such situations could not be conducted by the conventional approaches.

4 Conclusion

In this paper, we proposed a novel approach to analyze manipulability of manipulators including the effect of force. First, we derived required joint torques to compensate link gravity, payload, tip forces for keeping contact, and so on. Using operation range attached to every actuator, we formulated the relationship between the required joint torques and actually generable joint velocities. Using the relationship, we derived generable endeffector velocities from the actually generable joint velocities. The derived generable endeffector velocities can concern the effect of force exerted on the manipulator. We derived the corresponding manipulability measures: the volume of the generable endeffector velocity set, the maximum magnitude of endeffector velocity which is generable in any direction, and the maximum magnitude of generable endeffector velocity. In addition, we considered the evaluation when the forces exerted on the manipulators are not specified but the candidates (the set of possible exerted forces) are given. For example, it is true of the situation when a grinding task is given. To deal with this situation, we introduced required external force set (REFS), and presented how to derive manipulability. We also presented how to evaluate end effector force, taking required endeffector velocities into consideration. The validity of our approach was shown by numerical examples. This analysis can evaluate the cases which could not be evaluated by conventional manipulability approaches: for example, the case when gravity effect such as a payload has to be concerned, or the case when the manipulator contacts with an environment.

References

[1] Yoshikawa, T., 1990. *Foundations of Robotics*. MIT Press, Cambridge.

[2] Watanabe, T., and Yoshikawa, T., 2007. "Grasping optimization using a required external force set". *IEEE Transactions on Automation Science and Engineering,* **4**(1), pp. 52–66.

[3] Watanabe, T., and Yoshikawa, T., 2003. "Optimization of grasping an object by using required acceleration and equilibrium-force sets". *Proc. of IEEE/ASME International Conference on Advanced Intelligent Mechatronics*, pp. 338–343.

[4] Chiacchio, P., Chiaverini, S., Sciavicco, L., and Siciliano, B., 1991. "Global task space manipulability ellipsoids for multiple-arm systems". *IEEE Transactions on Robotics and Automation,* **7**(5), October, pp. 678–685. correspondence.

[5] Bicchi, A., Melchiorri, C., and Balluchi, D., 1995. "On the mobility and manipulability of general multiple limb robots". *IEEE Transactions on Robotics and Automation,* **11**(2), April, pp. 215–228.

[6] Bicchi, A., and Prattichizzo, D., 2000. "Manipulability of cooperating robots with unactuated joints and closed-chain mechanisms". *IEEE Transactions on Robotics and Automation,* **16**(4), August, pp. 336–345.

[7] Hara, K., Yokogawa, R., and Yokogawa, A., 1998. "A graphical method for evaluating static characteristics of the human finger by force manipulability". *Proc. of the IEEE International Conference on Robotics and Automation*, pp. 1623–1628.

[8] Fukuda, K., 2004. *Frequently Asked Questions in Polyhedral Computation*. [Online]. Available: `http://www.ifor.math.ethz.ch/~fukuda/polyfaq/polyfaq.html`.

[9] Fukuda, K., and Prodon, A., 1996. *Double description method revisited In M. Deza, R. Euler, and I. Manoussakis, editors,* Combinatorics and Computer Science*, volume 1120 of* Lecture Notes in Computer Science*, pages 91–111*. Springer-Verlag. [Online]. Available: `http://www.ifor.math.ethz.ch/~fukuda/cdd_home/index.html`.

[10] Fukuda, K., 1999. *cdd/cdd+ Reference Manual*. [Online]. Available: `http://www.ifor.math.ethz.ch/~fukuda/cdd_home/index.html`.

[11] Barber, C. B., Dobkin, D. P., and Huhdanpaa, H., 1996. "The quickhull algorithm for convex hulls". *ACM Transactions on Mathematical Software,* **22**(4), December, pp. 469–483.

List of Tables

List of Figures

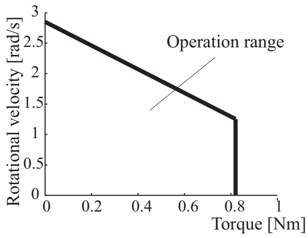


Fig. 1. Operation range of torque and velocity (maxon DC motor Amax16 (2W) with gear (ratio: 371:1)); supposing to control with nominal voltage

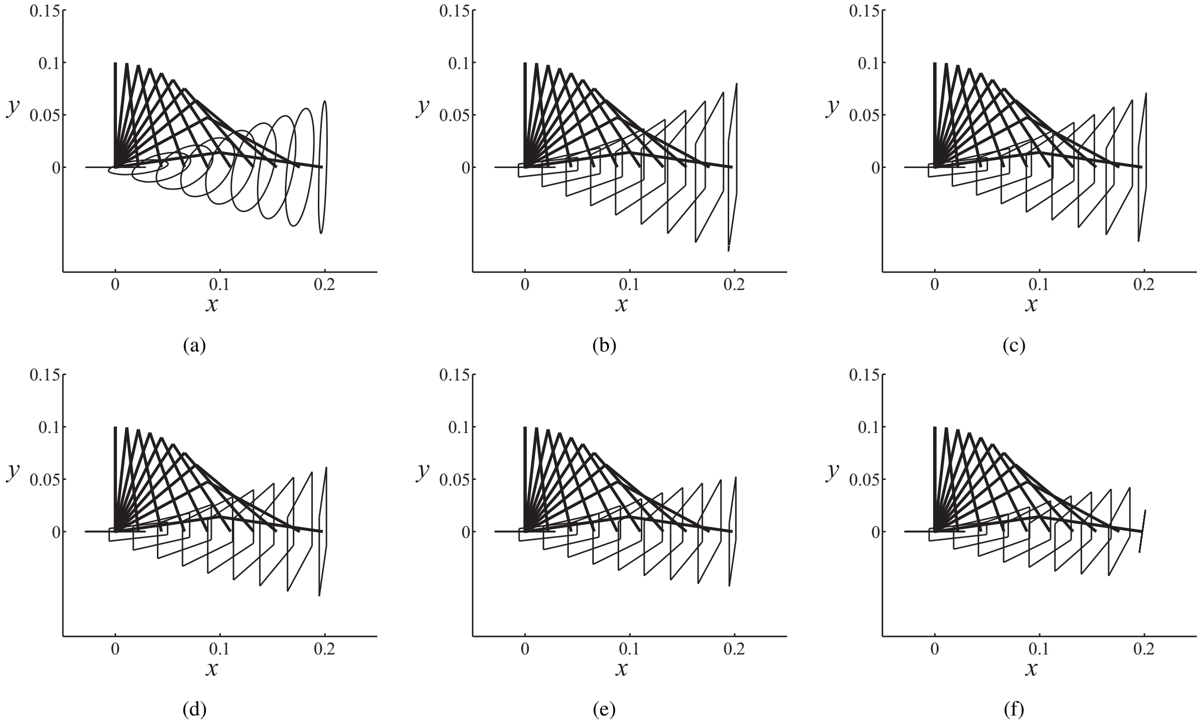
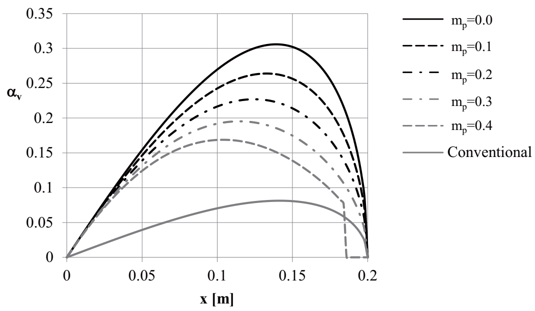
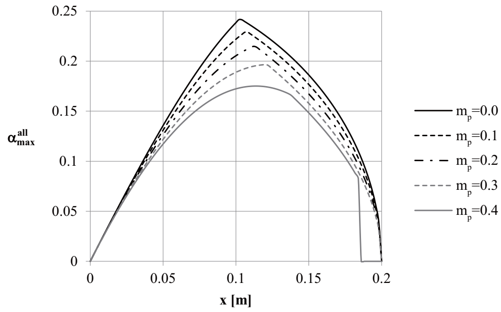


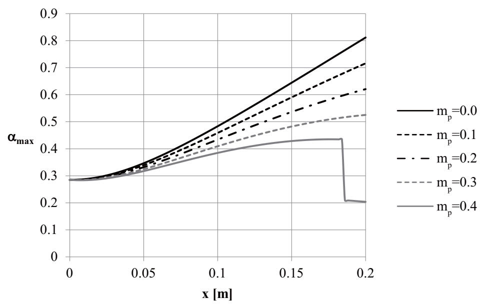
Fig. 2. Manipulability for 2-link planar manipulator. (a) original manipulability ellipsoid. (b)∼(f) new manipulability polyhedron (TVMP): (b) $m_p = 0.0$, (c) $m_p = 0.1$, (d) $m_p = 0.2$, (e) $m_p = 0.3$, and (f) $m_p = 0.4$.



(a) α_v



(b) α_{max}^{all}



(c) α_{max}

Fig. 3. Manipulability measures for 2-link planar manipulator

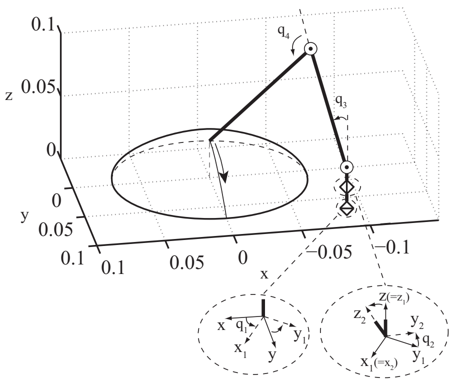


Fig. 4. Target system in 3 dimensional space

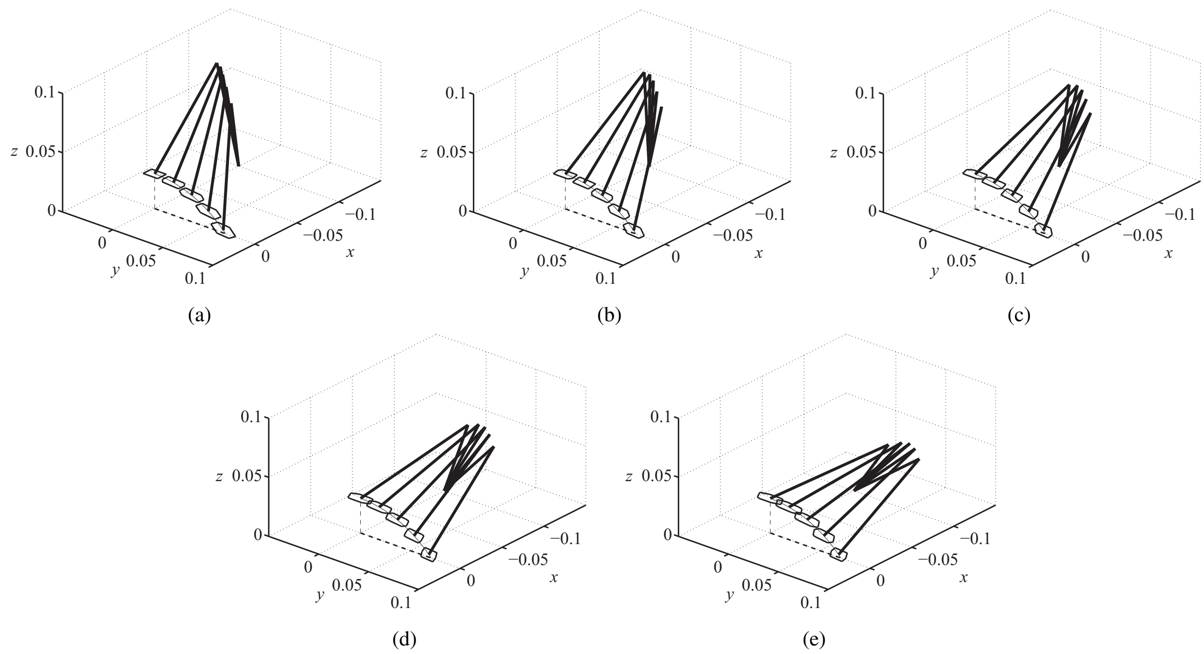
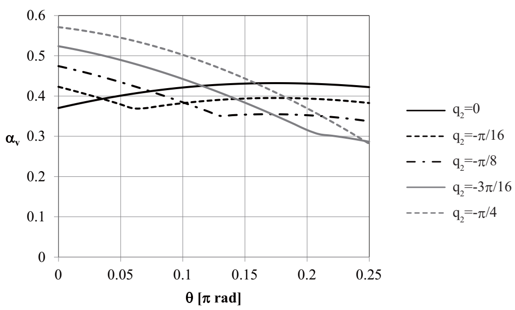
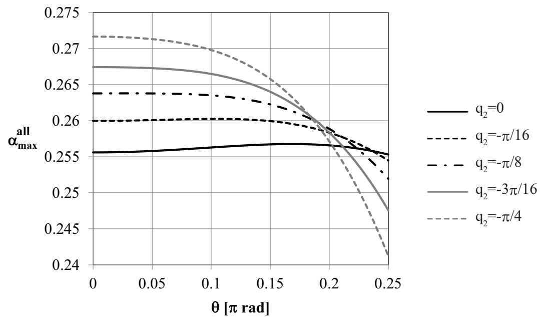


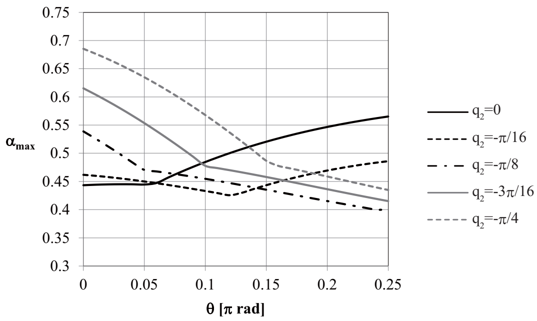
Fig. 5. New manipulability polyhedra (TVMP) for the manipulator shown in Fig.4: (a) $q_2 = 0$, (b) $q_2 = -\pi/16$, (c) $q_2 = -\pi/8$, (d) $q_2 = -3\pi/16$, and (e) $q_2 = -\pi/4$.



(a) α_v



(b) α_{max}^{all}



(c) α_{max}

Fig. 6. Manipulability measures for the manipulator shown in Fig.4: (a) α_v, (b) α_{max}^{all} and (c) α_{max}

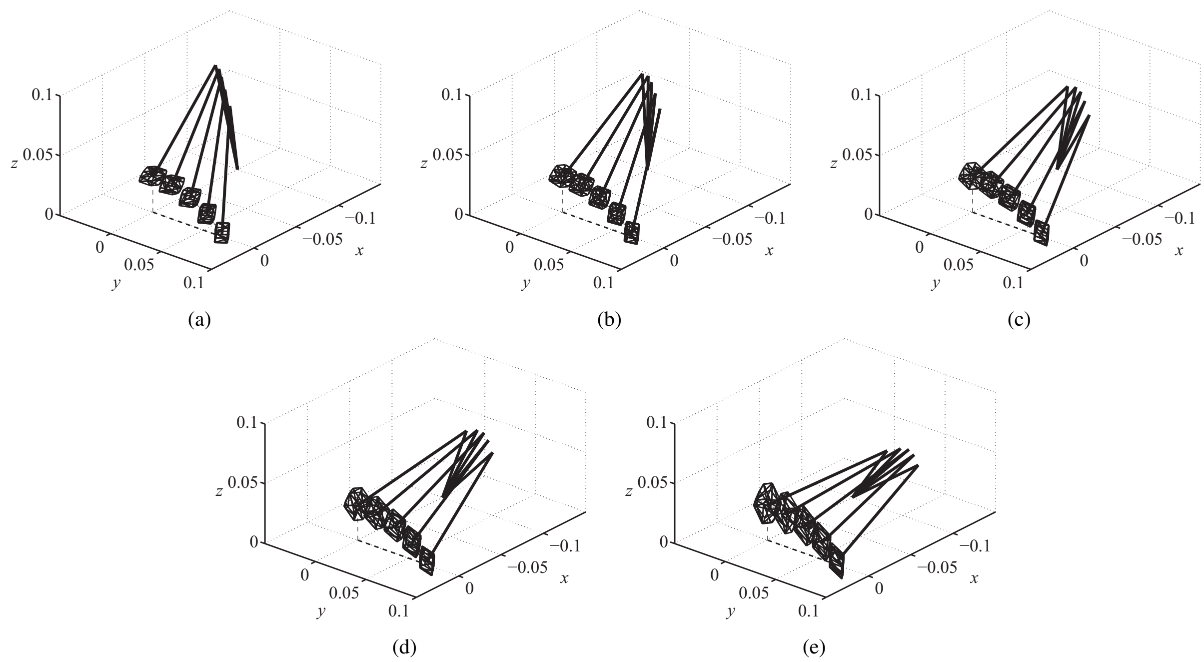
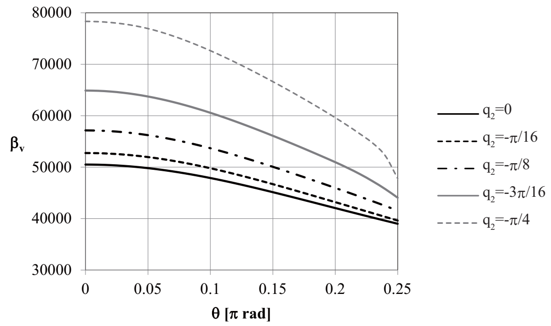
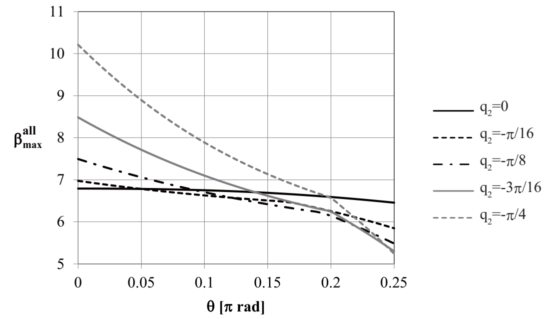


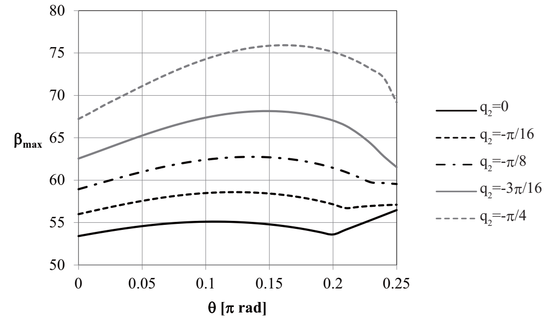
Fig. 7. New force manipulability polyhedra for the manipulator shown in Fig.4: (a) $q_2 = 0$, (b) $q_2 = -\pi/16$, (c) $q_2 = -\pi/8$, (d) $q_2 = -3\pi/16$, and (e) $q_2 = -\pi/4$.



(a) β_v



(b) β_{max}^{all}



(c) β_{max}

Fig. 8. Force manipulability measures for the manipulator shown in Fig.4: (a) β_v, (b) β_{max}^{all} and (c) β_{max}