Towards Whole Arm Manipulation by Contact State Transition

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Abstract— **This paper discusses the whole arm manipulation allowing the contact state transition. For manipulation of an object under fully constrained, the contact state transition becomes necessary. In order to realize the object manipulation, we first derive the feasible direction of the object manipulation by analyzing the active/passive closure properties for every combination of contact states. Second, we derive the set of joint torque to move the object in the feasible direction. These analyses also provide the joint torque to realize the manipulation at the planned contact states. Effectiveness of the proposed method is confirmed by some simulation results.**

I. Introduction

In the grasping and manipulation of an object, the two kind of important classes are introduced by Yoshikawa [1] to explicitly take into account the specific mechanism and capability of applying forces of the robotic hands, and also to remove the confusion in the definition of force closure [2]. One is active force closure where any arbitrary force and moment can be applied to a grasped object by fingers. The other is passive force closure where any arbitrary external force and moment exerted on a grasped object can be balanced without changing the pre-loaded joint torque and the motion of the object can be completely constrained. Under active force closure (such as fingertip grasps), the object can be manipulated in any directions, but the robustness is lost. Under passive force closure (corresponding to power grasps [3]), the object can not be manipulated, but can be robustly grasped. Some grasp configurations have both the active and passive closure properties. Such intermediate grasp configurations are named the hybrid active/passive closure grasps [1].

Fig.1 shows a 2D example of the whole arm manipulation [4] we have in mind. In this figure, the robotic hand first grasps an object firmly by utilizing the passive closure. Let us consider manipulating the grasped object from this fully constrained configuration. In order to manipulate an object arbitrarily, the robotic hand tries to change the grasp configuration to the active closure. However, to realize the active closure, the contact state between the finger and the object has to change since the object cannot move unless the passive closure is satisfied. The intermediate condition between the active and the passive closure corresponds to the hybrid closure grasp, and we can assume several grasp styles of hybrid closure grasp depending on the state of every contact point. During the object is grasped under the hybrid closure, the object may slip or roll on the surface of the finger. Also, the finger may detatch from the object. We can see from this example that, to realize this style of manipulation, the study of contact state transition under hybrid closure is necessary. More concretely, we have the following three research topics; 1) the feasible motion of the grasped object under given grasp configuration, 2) how to apply the joint torque to realize the change of contact conditions, and 3) calculation of the feasible path of contact state transition to finally realize the desired grasp configuration such as active closure. Among these problems, this paper aims to solve the first and second ones.

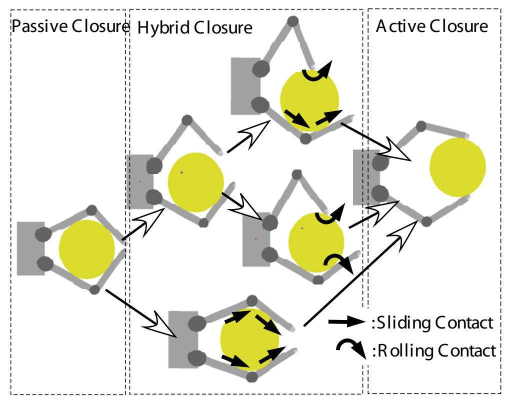


Fig. 1. Example of grasp transition from passive to active closure

In this paper, in order to realize such manipulation, we first classify the contact state and analyze the active and passive closure properties for every combination of contact states. In the analysis, we derive the feasible direction of the object manipulation for every combination of contact states. Second, we derive the set of joint torque enabling the object to move in the feasible direction and to achieve the change of contact conditions. The obtained results enable us to not only "plan the contact state transitions for desired object manipulation" but also "provide the joint torque to realize the manipulation at the planned contact states".

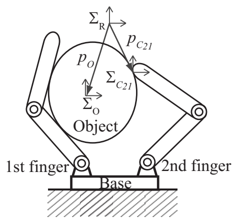


Fig. 2. Target System ($N = 2$)

A. Related Works

As described in the introduction, Trinkle et al. [5] pointed out that the confusion was introduced in the conventional definition of force closure. Yoshikawa [1] solved this confusion by redefining the force closure in the the active and the passive ones. As for the recent work of the active closure, Harada et al. analyzed the active force closure of multiple objects [6]. Also there are several works of formulating the general grasping system. Park et al. [7] derived contact forces and accelerations consistent with dynamics and friction law for a given torque-wrench pair. Bicchi et al. [8], [9], Wen et al. [10], and Park et al. [11] analyzed the manipulability of the general grasping systems. Harada et al. [12] presented a set of joint torque required to move the object in a desired direction. In the previous report, we showed the orthogonality between the active and the passive parts, and developed a control algorithm for manipulating an object under the hybrid closure [13], [14]. However, we have never shown the general framework of manipulation taking the contact state transitions into account.

On the other hand, Yashima and his colleagues have studied about the motion planning of an enveloped object grasped by multifingered hands with contact mode switching [15]–[18]. However, both detachment of the fingers and the passive property in a grasping system were not considered.

II. Target System and Definition

A. Target System

The target system is shown in Fig.2. In this paper, we consider a quasi-static object manipulation in the case where an arbitrary shaped rigid object is grasped by N fingers of a robotic hand. The nomenclatures are listed at appendix.

B. Definition

We consider manipulating the object taking the hybrid closure properties into account. Under hybrid closure, both the direction of the generalized force applied to the object and that of the object motion are constrained. We define the following two directions.

Direction of Active Force Closure (DAFC); The direction of the object's unit twist in which the fingers can do positive work is named the direction of active force closure (DAFC).

Direction of Passive Force Closure (DPFC); The direction of the object's unit wrench in which external wrench can be balanced without changing the joint torque is named the direction of passive force closure (DPFC).

Also, we will consider the transition of contact state. We define that the contact state changes among the following four states:
1) F-point : the contact point with static friction,
2) N-point : the contact point without friction,
3) S-point : the contact point with kinetic friction,
4) D-point : the point about to detach.

We note that the contact force is zero at D-points.

III. Basic Formulation of the System

A. Kinematics of the system

The relation of infinitesimal displacement among the contact points, the center of gravity of the object, and the joints of the fingers are expressed by the following two equations;

$$\Delta \boldsymbol{p}_{C_{F_{ij}}} = \boldsymbol{J}_{ij}\Delta \boldsymbol{q}_i, \quad \Delta \boldsymbol{p}_{C_{O_{ij}}} = \boldsymbol{G}_{ij}^T \Delta \boldsymbol{r} \tag{1}$$

where $\boldsymbol{J}_{ij} \in \mathcal{R}^{d\times M_i}$ denotes the Jacobian matrix and

$$\boldsymbol{G}_{ij} = \begin{bmatrix} \boldsymbol{I} \\ [(\boldsymbol{p}_{C_{O_{ij}}} - \boldsymbol{p}_o)\times] \end{bmatrix}.$$

Here, $\boldsymbol{I}$ represents an identity matrix, $[\boldsymbol{a}\times]$ represents a skew symmetric matrix equivalent to the cross product operation ($[\boldsymbol{a}\times]\boldsymbol{b} = \boldsymbol{a}\times\boldsymbol{b}$). From (1), we obtain

$$\boldsymbol{A}\begin{bmatrix} \Delta \boldsymbol{q} \\ \Delta \boldsymbol{r} \end{bmatrix} = \Delta \boldsymbol{p}_{C_F} - \Delta \boldsymbol{p}_{C_O} \triangleq -\Delta \boldsymbol{p}_C. \tag{2}$$

Next, to classify (2) according to the state of contact, we introduce the following selection matrices to derive the kinematic relation at every contact state.

$$\begin{aligned} \boldsymbol{H}_k &= \mathrm{diag}\begin{bmatrix} \boldsymbol{H}_{k_{ij}} \end{bmatrix} \in \mathcal{R}^{L_k\times Ld}(k = c, s, d), \quad (3)\\ \boldsymbol{H}_{c_{ij}} &= \begin{cases} \boldsymbol{I} & \text{for F-point} \\ \boldsymbol{n}_{ij}^T & \text{for S-point and N-point} \end{cases}, \\ \boldsymbol{H}_{s_{ij}} &= \boldsymbol{I} \quad \text{for S-point}, \\ \boldsymbol{H}_{d_{ij}} &= \boldsymbol{n}_{ij}^T \quad \text{for D-point}. \end{aligned}$$

Using the selection matrices, (2) becomes

$$\boldsymbol{A}_c \begin{bmatrix} \Delta \boldsymbol{q}^T & \Delta \boldsymbol{r}^T \end{bmatrix}^T = \boldsymbol{o}, \tag{4}$$

$$\boldsymbol{A}_d \begin{bmatrix} \Delta \boldsymbol{q}^T & \Delta \boldsymbol{r}^T \end{bmatrix}^T \leq \boldsymbol{o}, \tag{5}$$

$$\boldsymbol{A}_s \begin{bmatrix} \Delta \boldsymbol{q}^T & \Delta \boldsymbol{r}^T \end{bmatrix}^T = -\boldsymbol{p}_{Cs}, \tag{6}$$

where superscripts c, s and d expresses the corresponding state of contact (for example, $\boldsymbol{A}_d = \boldsymbol{H}_d\boldsymbol{A}$) and $\boldsymbol{o}$ denotes a zero vector.

By solving (4), we obtain

$$\begin{bmatrix} \Delta \boldsymbol{q} \\ \Delta \boldsymbol{r} \end{bmatrix} = \boldsymbol{\Lambda}\Delta\boldsymbol{\zeta} = \begin{bmatrix} \boldsymbol{\Lambda}_q \\ \boldsymbol{\Lambda}_r \end{bmatrix}\Delta\boldsymbol{\zeta} \tag{7}$$

where $\boldsymbol{\Lambda} \in \mathcal{R}^{(M+D)\times a}$ denotes an orthogonal matrix whose columns form bases of the null space of $\boldsymbol{A}_c$, $\Delta\boldsymbol{\zeta} \in \mathcal{R}^a$ is an arbitrary vector expressing the magnitude of each column of $\boldsymbol{\Lambda}$. Note that $\Delta\boldsymbol{\zeta}$ corresponds to the object and the finger motion within the constraint applied by the fingers.

B. Statics of the system

From (4) and the principle of virtual work, the following relation is obtained:

$$\begin{bmatrix} \boldsymbol{\tau} \\ -\boldsymbol{w} \end{bmatrix} = \boldsymbol{A}_c^T \boldsymbol{f}_c = \begin{bmatrix} \boldsymbol{J}_c^T \\ -\boldsymbol{G}_c \end{bmatrix} \boldsymbol{f}_c \tag{8}$$

From (8), the following relation is obtained;

$$\boldsymbol{f}_c = (\boldsymbol{J}_c^T)^{+}\boldsymbol{\tau} + \boldsymbol{\Gamma}\tilde{\boldsymbol{\xi}} = \boldsymbol{W}_f\, \boldsymbol{x} \tag{9}$$

where $(\boldsymbol{J}_c^T)^{+}$ denotes the pseudo-inverse matrix of $\boldsymbol{J}_c^T$ and $\boldsymbol{\Gamma} \in \mathcal{R}^{L_c \times \tilde{p}}$ is an orthogonal matrix whose columns form bases of the null space of $\boldsymbol{J}_c^T$, $\tilde{\boldsymbol{\xi}} \in \mathcal{R}^{\tilde{p}}$ denotes an arbitrary vector. Note that $\boldsymbol{\Gamma}\tilde{\boldsymbol{\xi}}$ expresses an internal force which makes no influence on the joint torque. By substituting (9) into (8), we get

$$\boldsymbol{w} = \boldsymbol{G}_c(\boldsymbol{J}_c^T)^{+}\boldsymbol{\tau} + \boldsymbol{G}_c\boldsymbol{\Gamma}\tilde{\boldsymbol{\xi}} \stackrel{\triangle}{=} \boldsymbol{D}\boldsymbol{\tau} + \boldsymbol{\Xi}\boldsymbol{\xi} \tag{10}$$

where $\boldsymbol{\Xi} \in \mathcal{R}^{D\times p}$ is an orthogonal matrix whose columns form bases of the $\boldsymbol{G}_c\boldsymbol{\Gamma}$, p is the rank of the $\boldsymbol{G}_c\boldsymbol{\Gamma}$, and $\boldsymbol{\xi} \in \mathcal{R}^{p}$ is an arbitrary vector expressing the magnitude of each column of $\boldsymbol{\Xi}$. Note that the second term in the right side of (10) can express a generable resultant force without changing the joint torque.

Here, we consider the tangential force component of $\boldsymbol{f}_{ij}$ at S-point, which isn't considered at the above discussion. $\boldsymbol{t}_{f_{ij}}$ is a kinetic friction force and can be expressed by

$$\boldsymbol{t}_{f_{ij}} = -b_{ij} n_{f_{ij}} \Delta\boldsymbol{p}_{C_{ij}}. \tag{11}$$

where b_{ij} (≥ 0) is a scalar value determined by frictional constraint. Then, the resultant force exerted on the object by $\boldsymbol{t}_{f_s}$ is given by

$$\boldsymbol{w}_s = \boldsymbol{G}_s\boldsymbol{t}_{f_s} = \boldsymbol{G}_s\boldsymbol{N}(\boldsymbol{f}_c, b_{ij})\Delta\boldsymbol{p}_{Cs}. \tag{12}$$

Note that $\boldsymbol{n}_{ij}^T\Delta\boldsymbol{p}_{C_{ij}} = \boldsymbol{o}$.

C. Frictional constraints

The frictional constraint for F-point can be represented by

$$\mathcal{F}_{fij} = \{\boldsymbol{f}_{ij} |\, |\boldsymbol{t}_{f_{ij}}| \leq \mu_{ij} n_{f_{ij}},\ n_{f_{ij}} \geq 0\}. \tag{13}$$

The frictional constraint for N-point and the normal component of S-point can be represented by

$$\mathcal{F}_{nij} = \{n_{f_{ij}} | n_{f_{ij}} \geq 0\}. \tag{14}$$

The frictional constraint for the tangential component of S-point can be represented by

$$\mathcal{F}_{sij} = \{\boldsymbol{t}_{f_{ij}} |\, |\boldsymbol{t}_{f_{ij}}| = \mu_{ij} n_{f_{ij}}\}. \tag{15}$$

Aggregating (13), (14) and (15) for all F-points, S-points, and N-points, we obtain

$$\begin{aligned} \mathcal{F} = \{&\boldsymbol{f}_c | \boldsymbol{f}_{ij} \in \mathcal{F}_{fij},\ ^{\forall}C_{ij} \in \text{F-point}, \\ &n_{f_{ij}} \in \mathcal{F}_{nij},\ ^{\forall}C_{ij} \in \text{N-point, S-point}\}, \end{aligned} \tag{16}$$

$$\mathcal{F}_s = \{\boldsymbol{t}_{f_s} | \boldsymbol{t}_{f_{ij}} \in \mathcal{F}_{sij},\ ^{\forall}C_{ij} \in \text{S-point}\}. \tag{17}$$

IV. FEASIBLE DIRECTION AND JOINT TORQUE FOR OBJECT MANIPULATION

In this section, we derive feasible direction of the object motion and the joint torques to realize the feasible object motion, in a grasping system under a hybrid closure. Also, we derive the feasible finger motion to realize a regrasping motion. From this analysis, we can obtain the feasible finger and object motion and the joint torques for the manipulation at every point of C-Space, when planning a manipulation with contact state transitions.

Since the feasible object motion corresponds to DAFC, we derive a set of DAFC with respect to arbitrary combination of contact states. Also, we derive DPFC. Next, we derive the joint torques to realize the object manipulation in the desired DAFC direction. Also, we derive the finger motion to regrasp the object without moving the object.

Let us consider the case where the all contact points are assigned into F-points, N-points, S-points, and D-points. Here, we consider the joint torque, $\boldsymbol{\tau}_{st}$, to grasp the object stably and make the object be in stationary state. From (10) and (16), $\boldsymbol{\tau}_{st}$ holds

$$-\boldsymbol{w}_{ex} = \boldsymbol{D}\boldsymbol{\tau}_{st} + \boldsymbol{\Xi}\boldsymbol{\xi}_{st} = \boldsymbol{G}_c\boldsymbol{W}_f\boldsymbol{x}_{st},\ \boldsymbol{W}_f\boldsymbol{x}_{st} \in \mathcal{F} \tag{18}$$

where $\boldsymbol{x}_{st}$ and $\boldsymbol{\xi}_{st}$, respectively, represent $\boldsymbol{x}$ and $\boldsymbol{\xi}$ in the case where the system is in a stationary state, $\boldsymbol{w}_{ex}$ denotes the external force such as gravitational force. Note that if there is no external forces exerted on the object, $\boldsymbol{w}_{ex} = \boldsymbol{o}$. Note also that we assume that $\boldsymbol{w}_s$ is not included in $\boldsymbol{w}_{ex}$.

A. DAFC

First, we consider the kinematic constraints for the object motion. Let $\tilde{\boldsymbol{\Lambda}}_r \in \mathcal{R}^{D\times\tilde{a}}$ be a full column rank matrix whose columns form orthonormal bases of the space spanned by column vectors of $\boldsymbol{\Lambda}_r$ in (7).

$$\begin{bmatrix} \boldsymbol{\Lambda}_q \\ \boldsymbol{\Lambda}_r \end{bmatrix} \Delta\boldsymbol{\zeta} = \begin{bmatrix} \tilde{\boldsymbol{\Lambda}}_q \\ \tilde{\boldsymbol{\Lambda}}_r \end{bmatrix} \Delta\tilde{\boldsymbol{\zeta}} + \begin{bmatrix} \tilde{\boldsymbol{\Lambda}}_{q_r} \\ \boldsymbol{O} \end{bmatrix} \Delta\tilde{\boldsymbol{\zeta}}_r \tag{19}$$

Here, the second term represents the redundancy of the joint variables which do not affect the object motion ($\tilde{\boldsymbol{\Lambda}}_{q_r} \in \mathcal{R}^{M\times(a-\tilde{a})}$, $\Delta\tilde{\boldsymbol{\zeta}}_r \in \mathcal{R}^{a-\tilde{a}}$). From (4), (5), (7), and (19), the allowable object and finger motions within the constraints are given by

$$\begin{aligned} &\mathcal{A}_r = \{\Delta\boldsymbol{r} | \Delta\boldsymbol{r} = \tilde{\boldsymbol{\Lambda}}_r\Delta\tilde{\boldsymbol{\zeta}} = \boldsymbol{\Phi}\Delta\boldsymbol{\zeta}_c,\ \Delta\boldsymbol{\zeta}_c \in \mathcal{A}\}, \\ &\mathcal{A}_q = \{\Delta\boldsymbol{q} | \Delta\boldsymbol{q} = \begin{bmatrix} \tilde{\boldsymbol{\Lambda}}_q & \tilde{\boldsymbol{\Lambda}}_{q_r} \end{bmatrix} \Delta\boldsymbol{\zeta}_c,\ \Delta\boldsymbol{\zeta}_c \in \mathcal{A}\}, \\ &\mathcal{A} = \{\Delta\boldsymbol{\zeta}_c | \boldsymbol{A}_d(\begin{bmatrix} \tilde{\boldsymbol{\Lambda}}_q & \tilde{\boldsymbol{\Lambda}}_{q_r} \\ \tilde{\boldsymbol{\Lambda}}_r & \boldsymbol{O} \end{bmatrix} \Delta\boldsymbol{\zeta}_c) \leq \boldsymbol{o}\}. \end{aligned} \tag{20}$$

The robot hand can do work on the object in the directions contained in the set $\mathcal{A}_r$.

Using (19), (6) becomes

$$\Delta\boldsymbol{p}_{Cs} = \boldsymbol{P}_s\Delta\boldsymbol{\zeta}_c,\ \Delta\boldsymbol{\zeta}_c \in \mathcal{A}. \tag{21}$$

Here, we introduce the following theorem which provides the generalized force set for DAFC.

Theorem 1: Suppose the object is grasped by $\boldsymbol{\tau}_{st}$ stasfying (18). Let $\boldsymbol{\tau}_{ad}$ be the joint torque added to $\boldsymbol{\tau}_{st}$ for doing work on the object. Let $\boldsymbol{x}_{ad}$ be $\boldsymbol{x}$ when $\boldsymbol{\tau}_{ad}$ is added to the system. DAFC is included in the following set:

$$\begin{aligned}\mathcal{D}_{DAFC} = \{\Delta\boldsymbol{r} | \Delta\boldsymbol{r} = \boldsymbol{\Phi}\Delta\boldsymbol{\zeta}_c,\ \Delta\boldsymbol{\zeta}_c \in \mathcal{A},\\ \boldsymbol{W}_f(\boldsymbol{x}_{ad}+\boldsymbol{x}_{st}) \in \mathcal{F},\ \boldsymbol{N}_t(\boldsymbol{y})\Delta\boldsymbol{\zeta}_c \in \mathcal{F}_s,\\ \Delta\boldsymbol{\zeta}_c^T(\boldsymbol{W}_1\boldsymbol{\tau}_{ad} + \boldsymbol{N}_1(\boldsymbol{y})\Delta\boldsymbol{\zeta}_c) > 0,\\ \boldsymbol{W}_2\boldsymbol{x}_{ad} + \boldsymbol{N}_2(\boldsymbol{y})\Delta\boldsymbol{\zeta}_c = \boldsymbol{o}\}\end{aligned} \quad (22)$$

Proof: First, from (9), (12), (16), (17) and (21), the frictional constraints are given by

$$\boldsymbol{f}_c = \boldsymbol{W}_f(\boldsymbol{x}_{ad}+\boldsymbol{x}_{st}) \in \mathcal{F}, \quad (23)$$

$$\boldsymbol{t}_{f_s} = \boldsymbol{N}(\boldsymbol{f}_c, b_{ij})\boldsymbol{P}_s\Delta\boldsymbol{\zeta}_c = \boldsymbol{N}_t(\boldsymbol{y})\Delta\boldsymbol{\zeta}_c \in \mathcal{F}_s. \quad (24)$$

Second, the allowable object motion is given by (20).

Third, when moving the object in the allowable direction, the work done by $\boldsymbol{\tau}_{ad}$ is given by

$$\begin{aligned}&\Delta\boldsymbol{r}^T\boldsymbol{w} = \Delta\boldsymbol{r}^T(\boldsymbol{D}\boldsymbol{x}_{ad} + \boldsymbol{\Xi}\boldsymbol{\xi}_{ad} + \boldsymbol{w}_s)\\ &= \Delta\boldsymbol{\zeta}_c^T(\boldsymbol{W}_1\boldsymbol{\tau}_{ad} + \boldsymbol{N}_1(\boldsymbol{y})\Delta\boldsymbol{\zeta}_c) + \Delta\tilde{\boldsymbol{\zeta}}^T\tilde{\boldsymbol{\Lambda}}_r^T\boldsymbol{\Xi}\boldsymbol{\xi}_{ad},\ \Delta\boldsymbol{\zeta}_c \in \mathcal{A}.\end{aligned} \quad (25)$$

Here, $\boldsymbol{\Xi}\boldsymbol{\xi}_{ad}$ is resultant force and moment which work to counteract external force and moment in the direction contained in $\mathrm{Im}(\boldsymbol{\Xi})$, only when the external force and moment are exerted on the object. Then, the applied force and moment by $\boldsymbol{\tau}_{ad}$ in the direction contained in $(\mathrm{Im}(\boldsymbol{D}) \cap \mathrm{Im}(\boldsymbol{\Xi}))$ will be counteracted. Here, we can easily show $\tilde{\boldsymbol{\Lambda}}_r^T\boldsymbol{\Xi} = \boldsymbol{O}$ and $\mathrm{rank}(\boldsymbol{\Xi}\ \ \tilde{\boldsymbol{\Lambda}}_r) = D$ by the similar way as our previous paper [13], [14], which corresponds to the orthogonality between DAFC and DPFC. Therefore, the counteracted force and moment is given by $(\boldsymbol{I} - \tilde{\boldsymbol{\Lambda}}_r\tilde{\boldsymbol{\Lambda}}_r^T)(\boldsymbol{D}\boldsymbol{\tau}_{ad} + \boldsymbol{w}_s)$ and the following relation holds

$$(\boldsymbol{I} - \tilde{\boldsymbol{\Lambda}}_r\tilde{\boldsymbol{\Lambda}}_r^T)(\boldsymbol{D}\boldsymbol{\tau}_{ad} + \boldsymbol{w}_s) = -\boldsymbol{\Xi}\boldsymbol{\xi}_{ad}. \quad (26)$$

From (20), (23), (24), (25), (26), and $\tilde{\boldsymbol{\Lambda}}_r^T\boldsymbol{\Xi} = \boldsymbol{O}$, DAFC is given by (22). ■

B. DPFC

The tangential component of the contact forces at S-points works not for counteracting external forces but for avoiding the object motion. Therefore, we do not regrad the directions as DPFC. Consequently, we only have to consider the contact forces at F-points and N-points, and the normal component of the contact forces at S-points.

DPFC is the directions of external force and moment which can be balanced without changing $\boldsymbol{\tau}_{st}$ in the state given by (18). Let be $\boldsymbol{w}_p$ be such external forces and moments. $\boldsymbol{w}_p$ holds the following relation.

$$-\boldsymbol{w}_p - \boldsymbol{w}_{ex} = \boldsymbol{G}_c(\boldsymbol{f}_{c_{st}} - \boldsymbol{f}_{c_p}) = \boldsymbol{D}\boldsymbol{\tau}_{st} + \boldsymbol{\Xi}(\boldsymbol{\xi}_{st} - \boldsymbol{\xi}_p) \quad (27)$$

Here, $\boldsymbol{f}_{c_p}$, $\tilde{\boldsymbol{\xi}}_p$, and $\boldsymbol{\xi}_p$, respectively, are $\boldsymbol{f}_c$, $\tilde{\boldsymbol{\xi}}$, and $\boldsymbol{\xi}$ corresponding to $\boldsymbol{w}_p$. If $\boldsymbol{f}_{c_{st}} - \boldsymbol{f}_{c_p} \in \mathcal{F}$, $\boldsymbol{\Xi}\boldsymbol{\xi}_p$ represents DPFC. Then, DPFC is given by

$$\mathcal{D}_{DPFC} = \{\boldsymbol{w}_p | \boldsymbol{w}_p = \boldsymbol{\Xi}\boldsymbol{\xi}_p = \boldsymbol{G}_c\boldsymbol{\Gamma}\tilde{\boldsymbol{\xi}}_p, \boldsymbol{f}_{c_{st}} - \boldsymbol{\Gamma}\tilde{\boldsymbol{\xi}}_p \in \mathcal{F}_f\}. \quad (28)$$

C. Joint torques to realize the object motion in DAFC

In this subsection, we derive the joint torques to move the object in a desired DAFC with the contact state transitions. Using $\tilde{\boldsymbol{\Lambda}}_{q_r}$ in (19), $\boldsymbol{\tau}$ can be represented by

$$\boldsymbol{\tau} = \boldsymbol{J}_c^T\boldsymbol{f}_c + \tilde{\boldsymbol{\Lambda}}_{q_r}\tilde{\boldsymbol{\tau}}. \quad (29)$$

The second term represents the redundancy of the joint torques which do not affect the contact forces applied to the object. Here, we consider deriving the joint torques corresponding to the first term.

In order to derive the joint torques for the desired object motion, the kinetic frictional forces at S-point is needed to be determined. But, when the fingers have the redundancy, the finger motion, namely, the kinetic frictional forces at S-point can not always be determined uniquely. Here, we determine the finger motion by optimizing a criterion function ϕ with respect to the redundancy component.

Let $\Delta\boldsymbol{r}_d = \tilde{\boldsymbol{\Lambda}}_r\Delta\tilde{\boldsymbol{\zeta}}_d$ be the unit vector and the desired $\Delta\boldsymbol{r} \in \mathcal{D}_{DAFC}$ given by (22). We set the following set:

$$\begin{aligned}\mathcal{S} = \{\Delta\hat{\boldsymbol{r}}_d = \tilde{\boldsymbol{\Lambda}}_r\Delta\tilde{\boldsymbol{\zeta}}_d \in \mathcal{D}_{DAFC},\ \Delta\boldsymbol{\zeta}_c = [\ \Delta\tilde{\boldsymbol{\zeta}}_d^T\ \ \Delta\tilde{\boldsymbol{\zeta}}_r^T\ ]^T\\ \boldsymbol{W}_f(\boldsymbol{x}_{ad}+\boldsymbol{x}_{st}) \in \mathcal{F},\ \boldsymbol{N}_t(\boldsymbol{y})\Delta\boldsymbol{\zeta}_c \in \mathcal{F}_s\\ \Delta\boldsymbol{\zeta}_c^T(\boldsymbol{W}_1\boldsymbol{\tau}_{ad} + \boldsymbol{N}_1(\boldsymbol{y})\Delta\boldsymbol{\zeta}_c) > 0,\\ \tilde{\boldsymbol{\Lambda}}_{q_r}^T\boldsymbol{\tau}_{ad} = \boldsymbol{o},\ \boldsymbol{W}_3\boldsymbol{x}_{ad} + \boldsymbol{N}_3(\boldsymbol{y})\Delta\boldsymbol{\zeta}_c = \boldsymbol{o}\}.\end{aligned} \quad (30)$$

Here, to specify the direction in which the fingers do work, we add the constraint in the direction $\tilde{\boldsymbol{\Lambda}}_r(\boldsymbol{I} - (\Delta\tilde{\boldsymbol{\zeta}}_d\Delta\tilde{\boldsymbol{\zeta}}_d^T)/(\sqrt{\tilde{\boldsymbol{\zeta}}_d^T\tilde{\boldsymbol{\zeta}}_d}))$.

Using the set $\mathcal{S}$, we consider the following problem.

$$\begin{array}{rl}\min_{\Delta\tilde{\boldsymbol{\zeta}}_r} & \phi\\ \text{subject to} & \mathcal{S}\end{array} \quad (31)$$

Solving this problem, the finger and the object motions ($\Delta\boldsymbol{\zeta}_{c_d}$) can be completely obtained. Then, the joint torques to realize the desired object and finger motions are given by

$$\mathcal{T} = \{\boldsymbol{\tau} | \boldsymbol{\tau} = \boldsymbol{\tau}_{ad} + \boldsymbol{\tau}_{st},\ \mathcal{S}|_{\Delta\boldsymbol{\zeta}_c = \Delta\boldsymbol{\zeta}_{c_d}}\}. \quad (32)$$

Note that $\boldsymbol{\tau}_{st}$ is added for keeping the grasping during the manipulation. If grasping is not needed during the manipulation (for example, dynamic manipulation), $\boldsymbol{\tau}_{st}$ can be set to zero in (32).

The frictional coefficient μ is labile. Therefore, we should deal with various μ in the derivation of the joint torques for the manipulation. At F-point, we only have to use the estimated smaller μ. However, at S-point, μ directly affects the kinetic frictional forces. Here, we consider the case where μ becomes $\nu\mu$ $(1 \geq \nu \geq 0)$ while from (32), the joint torques for the desired finger and object motions, $\boldsymbol{\tau}_\mu$, have been already obtained with respect to μ. Let $L_s^{'}$ be the number of S-points where the normal components of the contact forces induced by $\boldsymbol{\tau}_\mu$ are not zero. Let $n_{s\mu_i}$ $(i = 1, \cdots, L_s^{'})$ be the normal component of the contact force at each S-point induced by $\boldsymbol{\tau}_\mu$. If $L_s^{'} = 0$, the joint torques for the manipulation can be easily obtained by the linear combination of $\boldsymbol{\tau}_\mu$ and $\boldsymbol{\tau}_{st}$. If $L_s^{'} > 0$, the joint toruques can not be easily obtained for the nonliear constraints for the non-zero kinetic frictional forces. However, if the following equation can be solved subject to $\lambda_0 > 0$ and $\lambda_i \geq 0$, the joint toruques can be obtained by the linear combination $\lambda_0 \boldsymbol{\tau}_\mu + \Sigma_{i=1}^{L_s^{'}} \lambda_i \boldsymbol{\tau}_{st_i}$ where $\boldsymbol{\tau}_{st_i}$ denotes the $L_s^{'}$ sets of the joint torques at the steady state and $n_{sst_{ij}}$ denotes the normal component of the contact force at the S-point corresponding to $n_{s\mu_i}$, induced by $\boldsymbol{\tau}_{st_j}$ $(j = 1, \cdots, L_s^{'})$.

$$\begin{bmatrix} (\nu-1)n_{s\mu_1} & \nu n_{sst_{11}} & \cdots & \nu n_{sst_{1L_s^{'}}} \\ \vdots & \vdots & \ddots & \vdots \\ (\nu-1)n_{s\mu_{L_s^{'}}} & \nu n_{sst_{L_s^{'}1}} & \cdots & \nu n_{sst_{L_s^{'}L_s^{'}}} \\ 1 & 1 & \cdots & 1 \end{bmatrix} \begin{bmatrix} \lambda_0 \\ \vdots \\ \vdots \\ \lambda_{L_s^{'}} \end{bmatrix} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}$$

D. Finger motion for regrasping

Here we consider regrasping without moving the object. Letting $\Delta \boldsymbol{r}_d = \boldsymbol{o}$ in (30) and (32), the realizable finger motion can be obtained.

$$\begin{aligned} \mathcal{A}^{'} = \{ & \Delta \boldsymbol{q} | \Delta \boldsymbol{q} = \tilde{\boldsymbol{\Lambda}}_{q_r} \Delta \tilde{\boldsymbol{\zeta}}_r, \ \boldsymbol{J}_d \tilde{\boldsymbol{\Lambda}}_{q_r} \Delta \tilde{\boldsymbol{\zeta}}_r \leq \boldsymbol{o}, \\ & \begin{bmatrix} \boldsymbol{D} & \boldsymbol{G}_c \boldsymbol{\Gamma} \end{bmatrix} \boldsymbol{x} - \boldsymbol{G}_s \boldsymbol{N}(\boldsymbol{x}, b_{ij}) \boldsymbol{A}_s \tilde{\boldsymbol{\Lambda}}_{q_r} \Delta \tilde{\boldsymbol{\zeta}}_r = -\boldsymbol{w}_{ex}, \\ & \boldsymbol{W}_f \boldsymbol{x} \in \mathcal{F}, \ -\boldsymbol{N}(\boldsymbol{x}, b_{ij}) \boldsymbol{A}_s \tilde{\boldsymbol{\Lambda}}_{q_r} \Delta \tilde{\boldsymbol{\zeta}}_r \in \mathcal{F}_s \}. \end{aligned} \quad (33)$$

V. Numerical Examples

In order to verify our approach, we show some numerical examples. Consider the case shown in Fig.3. The frictional coefficient is set to 0.5 for all contact points. We set that $\boldsymbol{w}_{ex} = [0 \ -1 \ 0]^T$. The every contact state is denoted by "F"(F-point), "S"(S-point), and "D"(D-point). The contact states are expressed in the order of the number of contact point. For example, SFSD means C_1 is S-point, C_2 is F-point, C_3 is S-point, and C_4 is D-point. Using (22), we can know the feasible direction of the object manipulation for every combination of contact states. From the calculation, the following feasible combinations are obtained: FFDD, FFDS, FFSS, FSDS, FSSD, FSSS, SSSS. Note that the symmetric combinations of contact states are also feasible in the above list.

Here, we focus on FSDS combination of contact states. From (22), the feasible direction of the object manipulation (DAFC) is calculated as shown in Fig.4 (a). The schema of the obtained feasible object motion is shown in Fig.4 (b). From Fig.4, it can be seen that x direction and clockwise rotation around the rotation center shown in Fig.4 (b) are realizable.

Next, we consider the joint torques to move the object in $[1 \ 0.15 \ -1]^T$, as an example. From (32), the set of the joint torques is given by

$$\begin{aligned} & \mathcal{T} = \{ \Sigma_{j=1}^5 \lambda_j \boldsymbol{\tau}_j, \ \Sigma_{j=1}^5 \lambda_j = 1, \ \lambda_j \geq 0 \}, \\ & \boldsymbol{\tau}_1 = [-0.38 \ \ -0.17 \ \ 0.0093 \ \ 0]^T, \\ & \boldsymbol{\tau}_2 = [-0.31 \ \ 0.25 \ \ 0.43 \ \ 0]^T, \ \boldsymbol{\tau}_3 = [-0.38 \ \ -0.17 \ \ 0 \ \ 0]^T, \\ & \boldsymbol{\tau}_4 = [-0.15 \ \ 0.12 \ \ 0 \ \ 0]^T, \ \boldsymbol{\tau}_5 = [-0.31 \ \ 0.25 \ \ 0.43 \ \ 0]^T. \end{aligned}$$

Next, we consider the case shown in Fig.5. Here, we consider transiting the system from (1) to (4). In the transition from (1) to (2) and from (2) to (3), (33) is used. In the transition from (3) to (4), (30), (31) and (32) are used. We set $\phi = \sqrt{\Delta \boldsymbol{q}^T \boldsymbol{q}}$, $\mu = 0.5$, and $\boldsymbol{w}_{ex} = [0 \ \ 0 \ \ -1 \ \ 0 \ \ 0 \ \ 0]^T$. The every link is set to 1. The frictional cone is approximated by a 16-sided convex polyhedral cone.

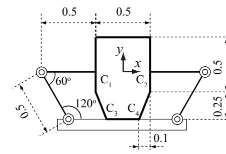
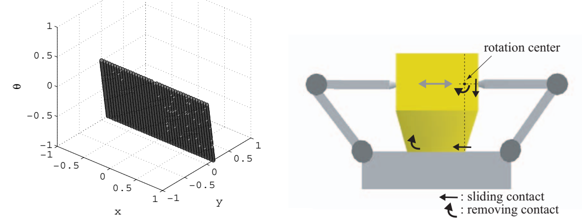


Fig. 3. Example



(a) DAFC (b) schema of DAFC

Fig. 4. DAFC and the schema of DAFC for FSDS combination of contact states

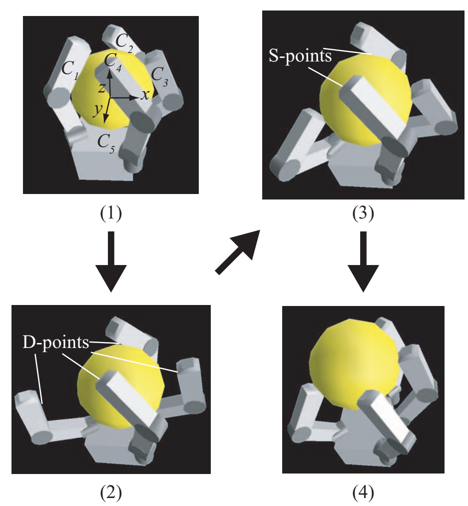


Fig. 5. Whole arm manipulation with several contact state transitions

At first, we consider transiting the system from (1) to (2). The combination of the contact states is DDDDF. Then, the realizable finger motion is given by

$$\begin{aligned} & \mathcal{A}^{'} = \{ \Delta \boldsymbol{q} | \ \boldsymbol{J}_d \Delta \boldsymbol{q} \leq \boldsymbol{o} \}, \ \boldsymbol{J}_d = \mathrm{diag}[\boldsymbol{J}_{di}] \ (i = 1 \sim 4), \\ & \boldsymbol{J}_{d1} = [-0.17 \ \ -0.88 \ \ 1.1 \ \ 0.39], \\ & \boldsymbol{J}_{d2} = [0.083 \ \ 0.51 \ \ 1.3 \ \ 0.47], \ \boldsymbol{J}_{d3} = [0 \ \ 0 \ \ 1 \ \ 0.5], \\ & \boldsymbol{J}_{d4} = [-0.083 \ \ -0.51 \ \ 1.3 \ \ 0.47]. \end{aligned}$$

Note that the finger motion for the transition from (2) to (3) is similarly obtained.

Next, we consider transiting the system from (3) to (4). The combination of the contact states is FSFSD. When we move the object in the direction $[0 \ 0 \ 1 \ 0 \ 0 \ 0]^T$, the joint torques for the manipulation is given by

$$\mathcal{T} = \{ \Sigma_{j=1}^2 \lambda_j \boldsymbol{\tau}_{v_j} + \Sigma_{i=1}^2 \beta_i \boldsymbol{\tau}_{w_i}, \ \Sigma_{j=1}^2 \lambda_j = 1, \ \lambda_j, \beta_i \geq 0 \},$$

$$\boldsymbol{\tau}_{v_j} = \mathrm{col}\left[\boldsymbol{\tau}_{v_{jk}}\right] \ (k = 1 \sim 4),\ \boldsymbol{\tau}_{v_{j2}} = \boldsymbol{\tau}_{v_{j4}} = \boldsymbol{o},\ ^{\forall} j,$$
$$\boldsymbol{\tau}_{w_j} = \mathrm{col}\left[\boldsymbol{\tau}_{w_{jk}}\right] \ (k = 1 \sim 4),\ \boldsymbol{\tau}_{w_{j2}} = \boldsymbol{\tau}_{w_{j4}} = \boldsymbol{o},\ ^{\forall} j,$$
$$\boldsymbol{\tau}_{v_{11}} = [-0.33\ \ -0.48\ \ -0.22\ \ -0.50]^T,$$
$$\boldsymbol{\tau}_{v_{21}} = [-0.28\ \ -0.41\ \ -0.22\ \ -0.50]^T,$$
$$\boldsymbol{\tau}_{v_{13}} = [0\ \ 0\ \ 0.72\ \ 0.29]^T,\ \boldsymbol{\tau}_{v_{23}} = [0\ \ 0\ \ 0.64\ \ 0.21]^T,$$
$$\boldsymbol{\tau}_{w_{11}} = [-0.28\ \ -0.41\ \ -0.22\ \ -0.50]^T,$$
$$\boldsymbol{\tau}_{w_{21}} = [-0.68\ \ -1.00\ \ 0.068\ \ -0.012]^T,$$
$$\boldsymbol{\tau}_{w_{13}} = [0\ \ 0\ \ 0.64\ 0.21]^T,\ \boldsymbol{\tau}_{w_{23}} = [0\ \ 0\ \ 1.00\ \ 1.15]^T.$$

From the above analysis, it can be seen that the motions of the fingers and the object shown in Fig.5 can be realized.

VI. CONCLUSION

In this paper we have discussed whole arm manipulation by taking the contact state transition into account. In order to realize the object manipulation, we have analyzed the active and passive closure properties in a hybrid closure grasp, for every combination of contact states. In the analysis, we have derived the feasible direction of the object manipulation for every combination of contact states. We further derived the set of joint torque to move the object in the feasible directions. Through some numerical examples, we showed that the grasp transition from the passive to active closure can be realized by using the proposed approach. In future, we plan to solve the path planning for object manipulation by considering the contact state transition.

APPENDIX

NOMENCLATURES

col	a column vector or matrix formed by the following elements.
diag	a block diagonal matrix.
N	Number of fingers.
M_i	Number of joints of the ith finger ($i = 1, 2, \cdots, N$).
L_i	Number of contact points on the ith finger.
M	Number of total joints ($= \Sigma_{i=1}^{N} M_i$).
L	Number of total contact points ($= \Sigma_{i=1}^{N} L_i$) .
L_k	Number of columns of selection matrix (k= c,s,d).
D	3/6 in 2/3 dimensional space.
d	2/3 in 2/3 dimensional space.
Σ_R	Reference coordinate frame.
Σ_O	Object coordinate frame fixed at the object.
C_{ij}	The jth contact point of the ith finger ($j = 1, 2, \cdots, L_i$).
$C_{F_{ij}}$	The contact point on the finger corresponding to C_{ij}.
$C_{O_{ij}}$	The contact point on the finger corresponding to C_{ij}.
Σ_{C_I}	Coordinate frame fixed at C_I ($I = C_{F_{ij}}, C_{O_{ij}}$).
$\boldsymbol{r}$:	Position and orientation of Σ_O ($\in \mathcal{R}^D$).
$\boldsymbol{q}_i$	Joint angle vector of the ith finger ($\in \mathcal{R}^{M_i}$).
$\Delta\boldsymbol{q}$	$=\mathrm{col}[\Delta\boldsymbol{q}_i] \in \mathcal{R}^M$.
$\boldsymbol{p}_I$	Position of the origin of Σ_I ($I = O, C_{F_{ij}}, C_{O_{ij}}, C_{ij}$) ($\in \mathcal{R}^d$).
$\Delta\boldsymbol{p}_J$	$=\mathrm{col}[\Delta\boldsymbol{p}_I] \in \mathcal{R}^{Ld}$ ($I = C_{F_{ij}}, C_{O_{ij}}, C_{ij},\ J = C_F, C_O, C$).
$\boldsymbol{n}_{ij}$	Unit normal vector (directing to the inward of the object) at C_{ij}.
$\boldsymbol{G}^T$	$=\mathrm{col}[\boldsymbol{G}_{ij}^T] \in \mathcal{R}^{D \times Ld}$.
$\boldsymbol{J}$	$= \mathrm{diag}\left[\begin{bmatrix}\boldsymbol{J}_{11}\\ \vdots\\ \boldsymbol{J}_{1L_1}\end{bmatrix} \begin{bmatrix}\boldsymbol{J}_{21}\\ \vdots\\ \boldsymbol{J}_{2L_2}\end{bmatrix} \cdots \begin{bmatrix}\boldsymbol{J}_{N1}\\ \vdots\\ \boldsymbol{J}_{NL_N}\end{bmatrix}\right] \in \mathcal{R}^{Ld \times M}$.
$\boldsymbol{A}$	$=[\ \boldsymbol{J}\quad -\boldsymbol{G}^T\ ] \in \mathcal{R}^{L_c \times (M+D)}$.
$\boldsymbol{w}$	Resultant force and moment applied to the object ($\in \mathcal{R}^D$).
$\boldsymbol{f}_{ij}$	Contact force vector ($\in \mathcal{R}^d$).
$\boldsymbol{f}$	$=\mathrm{col}[\boldsymbol{f}_{ij}] \in \mathcal{R}^{Ld}$. Note that $\boldsymbol{f}_k = \boldsymbol{H}_k \boldsymbol{f}$(k= c,s,d).
$\boldsymbol{\tau}$	Joint torque vector ($\in \mathcal{R}^M$).
$n_{f_{ij}}$	Normal force component of $\boldsymbol{f}_{ij}$.
$\boldsymbol{t}_{f_{ij}}$	$= [t_{f_{ij},1}\ \ t_{f_{ij},2}]^T$ where $t_{f_{ij},k}$ denotes the tangential force component of $\boldsymbol{f}_{ij}$ ($k = 1, 2$).
$\boldsymbol{t}_f$	$=\mathrm{col}[\boldsymbol{t}_{f_{ij}}]$.
μ_{ij}	Frictional coefficient at C_{ij}.
$\boldsymbol{x}$	$= [\boldsymbol{\tau}^T\quad \tilde{\boldsymbol{\xi}}^T]^T$.
$\boldsymbol{W}_f$	$= [(\boldsymbol{J}_c^T)^+\quad \boldsymbol{\Gamma}]$.
$\boldsymbol{N}$	$= \boldsymbol{N}(\boldsymbol{f}_c, b_{ij})$=diag $[-b_{ij} n_{f_{ij}}]$, $^{\forall} C_{ij} \in$ S-point.
$\boldsymbol{\Phi}$	$= [\tilde{\boldsymbol{\Lambda}}_r\quad \boldsymbol{O}]$.
$\Delta\boldsymbol{\zeta}_c$	$= [\Delta\tilde{\boldsymbol{\zeta}}^T\quad \Delta\tilde{\boldsymbol{\zeta}}_r^T]^T$.
$\boldsymbol{P}_s$	$= -\boldsymbol{A}_s \begin{bmatrix} \tilde{\boldsymbol{\Lambda}}_q & \tilde{\boldsymbol{\Lambda}}_{qr} \\ \tilde{\boldsymbol{\Lambda}}_r & \boldsymbol{O} \end{bmatrix}$.
$\boldsymbol{y}$	$= [\boldsymbol{x}_{ad}^T\ \boldsymbol{x}_{st}^T\ b_{11}\ \cdots]^T$.
$\boldsymbol{N}_t(\boldsymbol{y})$	$= \boldsymbol{N}(\boldsymbol{x}_{ad}, \boldsymbol{x}_{st}, b_{ij})\boldsymbol{P}_s$.
$\boldsymbol{W}_1$	$= \boldsymbol{\Phi}^T \boldsymbol{D}$.
$\boldsymbol{W}_2$	$= [(\boldsymbol{I} - \tilde{\boldsymbol{\Lambda}}_r \tilde{\boldsymbol{\Lambda}}_r^T)\boldsymbol{D}\quad \boldsymbol{G}_c \boldsymbol{\Gamma}]$.
$\boldsymbol{W}_3$	$= \begin{bmatrix} (\boldsymbol{I} - \tilde{\boldsymbol{\Lambda}}_r \tilde{\boldsymbol{\Lambda}}_r^T)\boldsymbol{D} & \boldsymbol{G}_c \boldsymbol{\Gamma} \\ (\boldsymbol{I} - (\Delta\tilde{\boldsymbol{\zeta}}_d \Delta\tilde{\boldsymbol{\zeta}}_d^T)/(\sqrt{\tilde{\boldsymbol{\zeta}}_d^T \tilde{\boldsymbol{\zeta}}_d}))\tilde{\boldsymbol{\Lambda}}_r^T \boldsymbol{D} & \boldsymbol{O} \end{bmatrix}$.
$\boldsymbol{N}_1(\boldsymbol{y})$	$= \boldsymbol{\Phi}^T \boldsymbol{G}_s \boldsymbol{N}_t(\boldsymbol{y})$.
$\boldsymbol{N}_2(\boldsymbol{y})$	$= (\boldsymbol{I} - \tilde{\boldsymbol{\Lambda}}_r \tilde{\boldsymbol{\Lambda}}_r^T)\boldsymbol{G}_s \boldsymbol{N}_t(\boldsymbol{y})$.
$\boldsymbol{N}_3(\boldsymbol{y})$	$= (\boldsymbol{I} - (\Delta\tilde{\boldsymbol{\zeta}}_d \Delta\tilde{\boldsymbol{\zeta}}_d^T)/(\sqrt{\tilde{\boldsymbol{\zeta}}_d^T \tilde{\boldsymbol{\zeta}}_d}))\tilde{\boldsymbol{\Lambda}}_r^T \boldsymbol{G}_s \boldsymbol{N}_t(\boldsymbol{y})$.

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